

# Tutorial 2: Forward Kinematics

These questions are from the Practice Exercises of the Modern Robotics book. The solutions can be found on the book website. Please try your best before referring to the solutions. You should understand how to solve the problems.

## Question 1: KUKA LBR iiwa 7R robot arm

Figure 1 shows the KUKA LBR iiwa (LBR = “Leichtbauroboter,” German for lightweight robot; iiwa = “intelligent industrial work assistant”) 7R robot arm. The figure defines an  $\{s\}$  frame at the base with the  $\hat{y}_s$ -axis pointing out of the page and a  $\{b\}$  frame aligned with  $\{s\}$  at the end-effector. The robot is at its home configuration. The screw axes for the seven joints are illustrated (positive rotation about these axes is by the right-hand rule). The axes for joints 2, 4, and 6 are aligned, and the axes for joints 1, 3, 5, and 7 are identical at the home configuration.

Write  $M$  ( $T_{sb}$  when the robot is at its home configuration), the screw axes  $\mathcal{S}_1, \dots, \mathcal{S}_7$  in  $\{s\}$ , and the screw axes  $\mathcal{B}_1, \dots, \mathcal{B}_7$  in  $\{b\}$ .

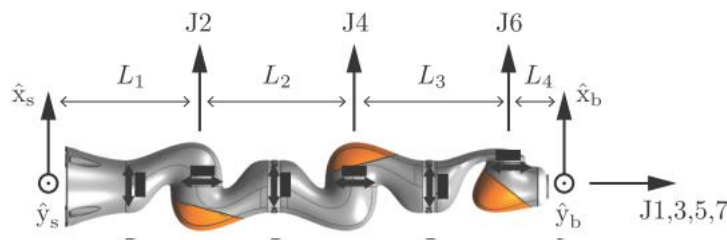


Figure 1. The KUKA LBR iiwa 7-dof robot. (Source: Modern Robotics)

## Question 2: KINOVA ultra lightweight 4-dof robot arm

Figure 2 shows a KINOVA ultra lightweight 4-dof robot arm at its home configuration. An  $\{s\}$  frame is at its base and a  $\{b\}$  frame is at its end-effector. All the relevant dimensions are shown. The  $\hat{y}_b$ -axis is displaced from the  $\hat{y}_s$ -axis by 9.8 mm, as shown in the image. Positive rotation about joint axis 1 is about the  $\hat{y}_s$ -axis (by the right-hand rule, as always) and joint axis 4 is about the  $\hat{y}_b$ -axis. Joint axes 2 and 3 are also illustrated.

- Write  $M$  (i.e.,  $T_{sb}$  when the robot is at its home configuration). All entries should be numerical (no symbols or math).
- Write the space-frame screw axes  $\mathcal{S}_1, \dots, \mathcal{S}_4$ . All entries should be numerical (no symbols or math).
- Give the product of exponentials formula for  $T_{sb}(\theta)$  for arbitrary joint angles  $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$ . Your answer should be purely symbolic (no numbers), using only the symbols  $M, \mathcal{S}_1, \dots, \mathcal{S}_4, \theta_1, \theta_2, \theta_3, \theta_4$ , and the matrix exponential.

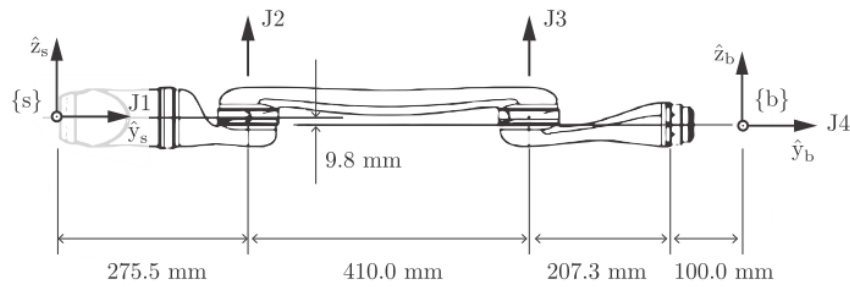


Figure 2. The KINOVA ultra lightweight 4-dof robot arm at its home configuration. (Source: Modern Robotics)

### Question 3: Sawyer collaborative robot

Figures 3 and 4 show a Sawyer collaborative robot in action on a factory floor. This is a 7-dof robotic arm.

- Draw a stick and cylinder model of Sawyer (similar to the examples in Chapter 4), clearly showing all links and joints.
- Assuming the home configuration is shown in Figure 4, write the  $M$  matrix.
- Write the space-frame and body-frame screw axes for this robot.
- What is the end-effector position when the joints are set to  $(0, \frac{\pi}{2}, 0, \frac{\pi}{2}, 0, \frac{\pi}{2}, 0)$ ? (Hint: You might find the functions in the MR library to be useful, or may use create simple spreadsheet cells to help with matrix computation).



Figure 3. A Sawyer robot. (Source: Modern Robotics)

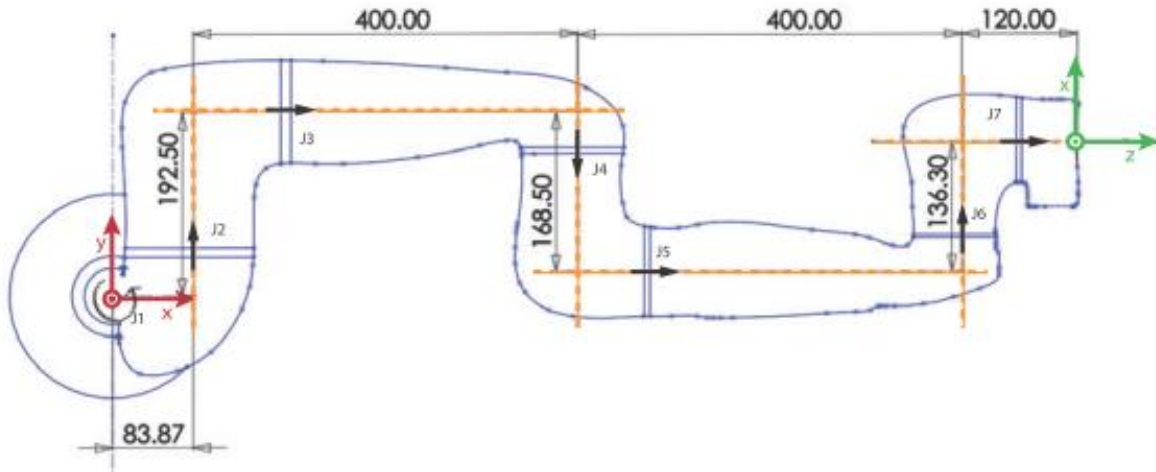


Figure 4. A top view of the Sawyer robot arm at its home configuration. Dimensions are in mm. Assume that the centerlines shown are the screw axes of the revolute joints. The  $\{s\}$  frame is at the base of the arm. The height from the base to the first joint is 317 mm. Note the joint axes are marked J1 to J7 on the diagram. (Source: Modern Robotics)

## Question 4: da Vinci Xi manipulator arm

Figure 5 shows a da Vinci Xi, used in several types of robot-assisted surgery. Though it is mechanically constrained to have only 3 degrees of freedom per arm, for the sake of this exercise assume each arm is a simple serial chain with 6 degrees of freedom.

- Write the  $M$  matrix for the arm if its home configuration is shown in Figure 6.
- Find the space frame screw axes for this system.
- Determine the position of the end-effector if the joints are at  $\left(0, \frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{2}\right)$ . Again, the MR Library will prove useful here.



Figure 5. Da Vinci Xi surgical robot. (Source: Modern Robotics)

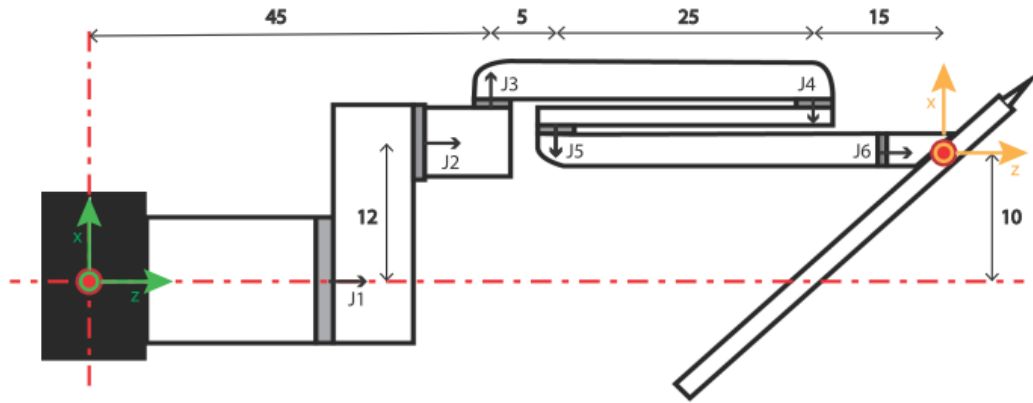


Figure 6. Top view of one da Vinci Xi surgical robot arm. Note that the grey regions represent R joints, green indicates the  $\{s\}$  frame, and yellow represents the end-effector frame  $\{b\}$  in this exercise. Dimensions are in cm. (Source: Modern Robotics)