Tutorial 2: Forward Kinematics

These questions are from the Practice Exercises of the Modern Robotics book. The solutions can be found on the book website. Please try your best before referring to the solutions. You should understand how to solve the problems.

Question 1: KUKA LBR iiwa 7R robot arm

Figure 1 shows the KUKA LBR iiwa (LBR = "Leichtbauroboter," German for lightweight robot; iiwa = "intelligent industrial work assistant") 7R robot arm. The figure defines an $\{s\}$ frame at the base with the $\hat{y}_s$-axis pointing out of the page and a $\{b\}$ frame aligned with $\{s\}$ at the end-effector. The robot is at its home configuration. The screw axes for the seven joints are illustrated (positive rotation about these axes is by the right-hand rule). The axes for joints 2, 4, and 6 are aligned, and the axes for joints 1, 3, 5, and 7 are identical at the home configuration.

Write M (T_{sb} when the robot is at its home configuration), the screw axes $\mathcal{S}_1, \cdots, \mathcal{S}_7$ in $\{s\}$, and the screw axes $\mathcal{B}_1, \cdots, \mathcal{B}_7$ in $\{b\}$.

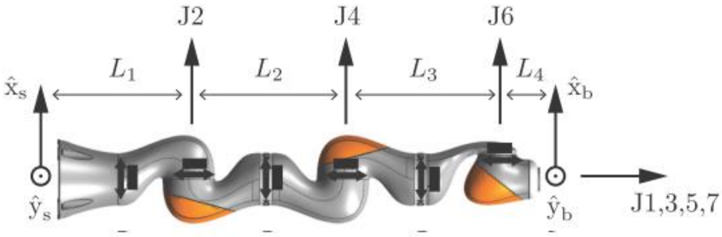


Figure 1. The KUKA LBR iiwa 7-dof robot. (Source: Modern Robotics)

Question 2: KINOVA ultra lightweight 4-dof robot arm

Figure 2 shows a KINOVA ultra lightweight 4-dof robot arm at its home configuration. An $\{s\}$ frame is at its base and a $\{b\}$ frame is at its end-effector. All the relevant dimensions are shown. The $\hat{y}_b$-axis is displaced from the $\hat{y}_s$-axis by 9.8 mm, as shown in the image. Positive rotation about joint axis 1 is about the $\hat{y}_s$-axis (by the right-hand rule, as always) and joint axis 4 is about the $\hat{y}_b$-axis. Joint axes 2 and 3 are also illustrated.

a. Write M (i.e., T_{sb} when the robot is at its home configuration). All entries should be numerical (no symbols or math).
b. Write the space-frame screw axes $\mathcal{S}_1, \cdots, \mathcal{S}_4$. All entries should be numerical (no symbols or math).
c. Give the product of exponentials formula for $T_{sb}(\theta)$ for arbitrary joint angles $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$. Your answer should be purely symbolic (no numbers), using only the symbols $M, \mathcal{S}_1, \cdots, \mathcal{S}_4, \theta_1, \theta_2, \theta_3, \theta_4$, and the matrix exponential.

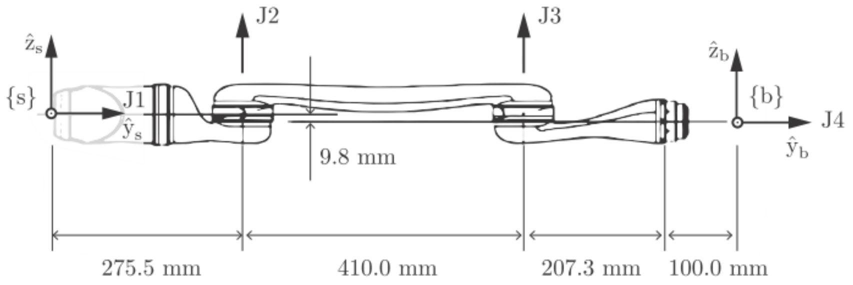


Figure 2. The KINOVA ultra lightweight 4-dof robot arm at its home configuration. (Source: Modern Robotics)

Question 3: Sawyer collaborative robot

Figures 3 and *4* show a Sawyer collaborative robot in action on a factory floor. This is a 7-dof robotic arm.

a. Draw a stick and cylinder model of Sawyer (similar to the examples in Chapter 4), clearly showing all links and joints.
b. Assuming the home configuration is shown in *Figure 4*, write the M matrix.
c. Write the space-frame and body-frame screw axes for this robot.
d. What is the end-effector position when the joints are set to $\left(0, \frac{\pi}{2}, 0, \frac{\pi}{2}, 0, \frac{\pi}{2}, 0\right)$? (Hint: You might find the functions in the MR library to be useful, or may use create simple spreadsheet cells to help with matrix computation).

Figure 3. A Sawyer robot. (Source: Modern Robotics)

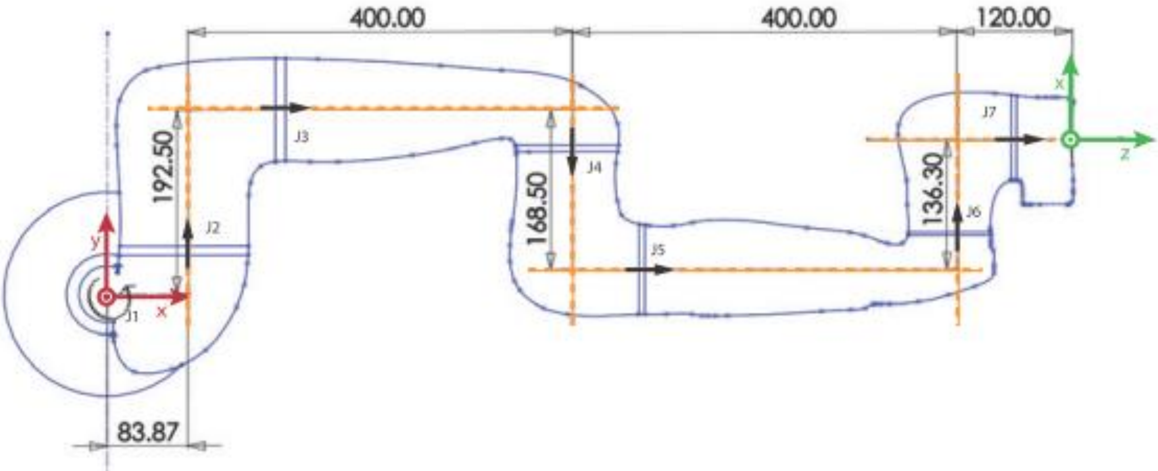


Figure 4. A top view of the Sawyer robot arm at its home configuration. Dimensions are in mm. Assume that the centerlines shown are the screw axes of the revolute joints. The $\{s\}$ frame is at the base of the arm. The height from the base to the first joint is 317 mm. Note the joint axes are marked J1 to J7 on the diagram. (Source: Modern Robotics)

Question 4: da Vinci Xi manipulator arm

Figure 5 shows a da Vinci Xi, used in several types of robot-assisted surgery. Though it is mechanically constrained to have only 3 degrees of freedom per arm, for the sake of this exercise assume each arm is a simple serial chain with 6 degrees of freedom.

a. Write the M matrix for the arm if its home configuration is shown in *Figure 6*.
b. Find the space frame screw axes for this system.
c. Determine the position of the end-effector if the joints are at $\left(0, \frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{2}\right)$. Again, the MR Library will prove useful here.

Figure 5. Da Vinci Xi surgical robot. (Source: Modern Robotics)

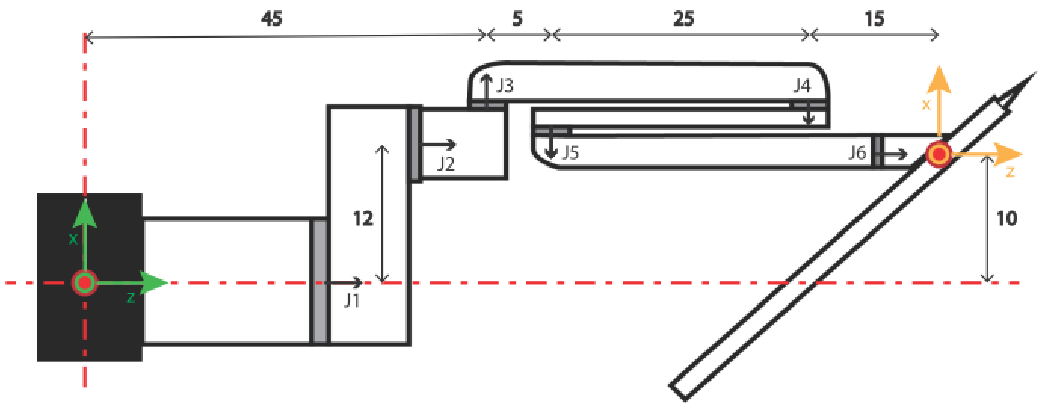
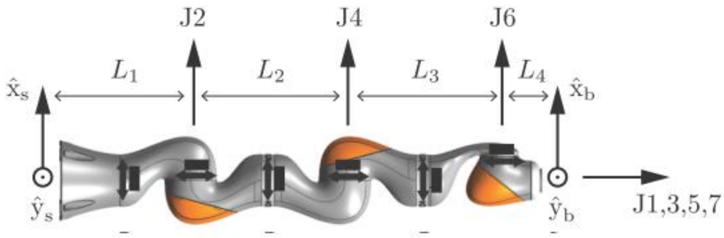


Figure 6. Top view of one da Vinci Xi surgical robot arm. Note that the grey regions represent R joints, green indicates the $\{s\}$ frame, and yellow represents the end-effector frame $\{b\}$ in this exercise. Dimensions are in cm. (Source: Modern Robotics)

Sample solution

Ref: Modern Robotics Practice Exercises

Question 1: KUKA LBR iiwa 7R robot arm



We can see that {b} at home configuration as shown in the figure has the same orientation with {s}. This gives a rotation matrix of identity matrix, $R_{sb} = I$. The translation is at a distance of $L_1 + L_2 + L_3 + L_4$ in the direction of $\hat{z}_s$ giving a translation of $t_{sb} = [0,0, L_1 + L_2 + L_3 + L_4]^T$.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 + L_2 + L_3 + L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For space/base form screw axes,

The direction of screw axes J1, 3, 5, 7 are along the $\hat{z}_s$ giving $\omega_1 = \omega_3 = \omega_5 = \omega_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. When rotating around these axes, there is no linear motion induced on the origin of {s} giving $v_1 = v_3 = v_5 = v_7 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

The direction of screw axes J2, 4, 6 are along the $\hat{x}_s$ giving $\omega_2 = \omega_4 = \omega_6 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. When rotationg around these axes, linear motion is induced on the origin of {s} along the direction y_s according to RHR. The magnitude of the linear velocity is equivalent to the distance from the axis to the origin of {s}. We have

$$v_2 = \begin{bmatrix} 0 \\ L_1 \\ 0 \end{bmatrix} \qquad v_4 = \begin{bmatrix} 0 \\ L_1 + L_2 \\ 0 \end{bmatrix} v_6 = \begin{bmatrix} 0 \\ L_1 + L_2 + L_3 \\ 0 \end{bmatrix}$$

Lining up the screw axes as columns, we get

$$\mathcal{S}_{\text{list}} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_1 & 0 & L_1 + L_2 & 0 & L_1 + L_2 + L_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

For body form screw axes (note with reference to {b}),

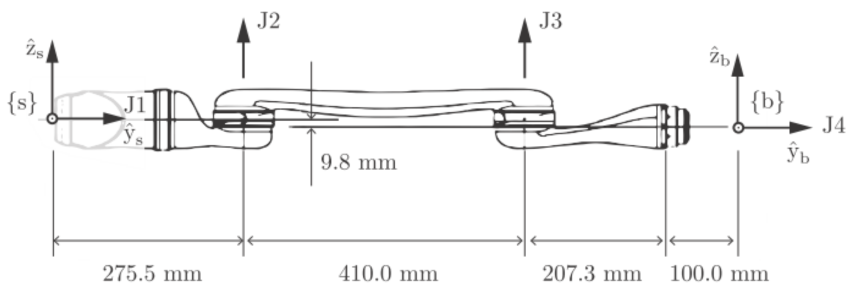
The direction of screw axes J1, 3, 5, 7 are along the $\hat{z}_b$ giving $\omega_1 = \omega_3 = \omega_5 = \omega_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. When rotating around these axes, there is no linear motion induced on the origin of {b} giving $v_1 = v_3 = v_5 = v_7 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

The direction of screw axes J2, 4, 6 are along the $\hat{x}_b$ giving $\omega_2 = \omega_4 = \omega_6 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. When rotating around these axes, linear motion is induced on the origin of {b} along the direction $-\hat{y}_b$ according to RHR. The magnitude of the linear velocity is equivalent to the distance from the axis to the origin of {b}. We have

$$v_2 = \begin{bmatrix} 0 \\ -(L_2 + L_3 + L_4) \\ 0 \end{bmatrix} \qquad v_4 = \begin{bmatrix} 0 \\ -(L_3 + L_4) \\ 0 \end{bmatrix} \qquad v_6 = \begin{bmatrix} 0 \\ -L_4 \\ 0 \end{bmatrix}$$

$$\mathcal{B}_{\text{list}} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(L_2 + L_3 + L_4) & 0 & -(L_3 + L_4) & 0 & -L_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Question 2: KINOVA ultra lightweight 4-dof robot arm



(a)

We can see that {b} at home configuration as shown in the figure has the same orientation with {s}. This gives a rotation matrix of identity matrix, $R_{sb} = I$. The translation is at a distance of $275.5 + 410.0 + 207.3 + 100.0 = 992.8\ mm$ in the direction of $\hat{y}_s$ and $9.8\ mm$ in the $-\hat{z}_s$ direction giving a translation of $t_{sb} = [0, 992.8, -9.8]^T$.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 992.8 \\ 0 & 0 & 1 & -9.8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b)

For space/base form screw axes,

The direction of screw axes J1, 4 are along the $\hat{y}_s$ giving $\omega_1 = \omega_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. When rotating around J1, there is no linear motion induced on the origin of {s} giving $v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. When rotating around J4, linear motion is induced on the origin of {s} along $\hat{x}_s$ direction according to RHR. The linear velocity is equivalent to the perpendicular distance from the axis to the origin of {s} giving $v_1 = \begin{bmatrix} 9.8 \\ 0 \\ 0 \end{bmatrix}$.

The direction of screw axes J2, 3 are along the $\hat{z}_s$ giving $\omega_2 = \omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. When rotating around these axes, linear motion is induced on the origin of {s} along the direction $\hat{x}_s$ according to RHR. The magnitude of the linear velocity is equivalent to the distance from the axis to the origin of {s}. We have

$$v_2 = \begin{bmatrix} 275.5 \\ 0 \\ 0 \end{bmatrix} \quad v_4 = \begin{bmatrix} 275.5 + 410.0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 685.5 \\ 0 \\ 0 \end{bmatrix}$$

Lining up the screw axes as columns of a matrix,

$$\mathcal{S}_{\text{list}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 275.5 & 685.5 & 9.8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

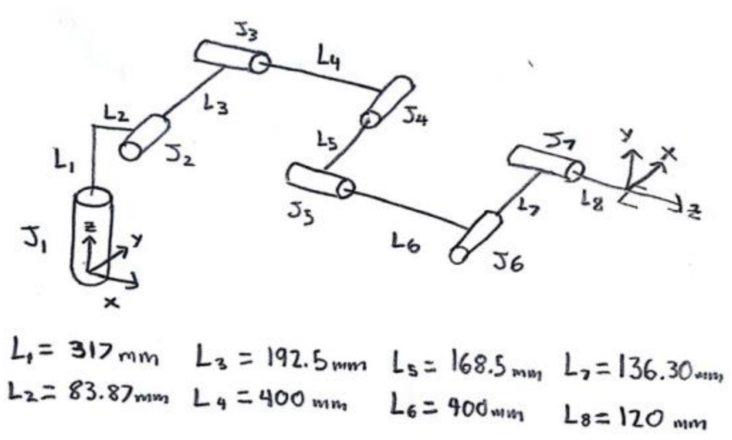
(c)

Product of exponential FK is given by $T_{sb}(\theta) = e^{[\mathcal{S}_1]\theta_1} \cdots e^{[\mathcal{S}_{n-1}]\theta_{n-1}} e^{[\mathcal{S}_n]\theta_n} M$. For this problem, there are four joints $n = 4$. We have

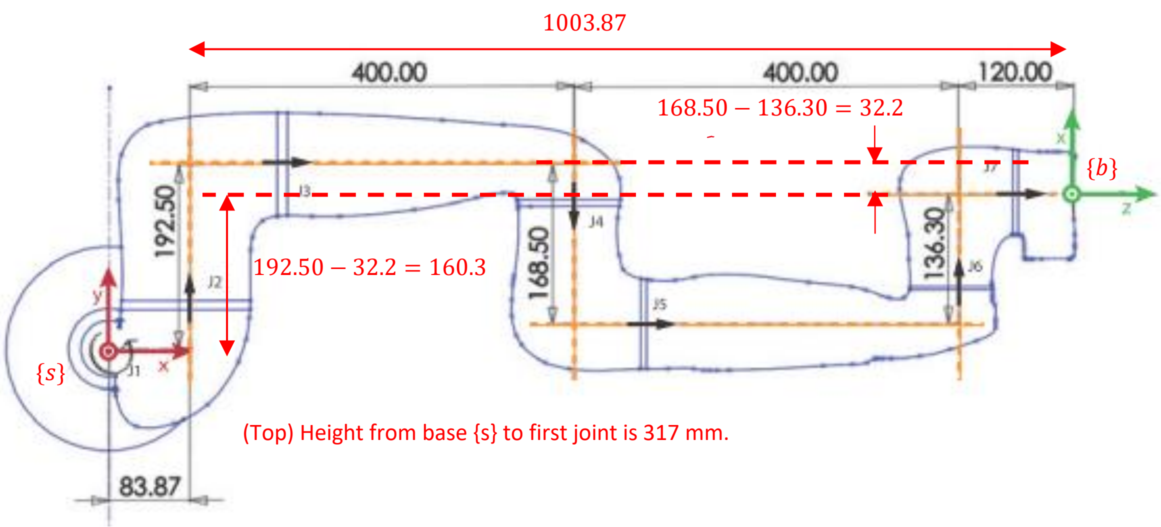
$$T_{sb}(\theta) = e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} e^{[\mathcal{S}_3]\theta_3} e^{[\mathcal{S}_4]\theta_4} M$$

Question 3: Sawyer collaborative robot

a.



b.



We can see that {b} at home configuration as shown in the figure has its $\hat{x}_b$ along $\hat{y}_s$, $\hat{y}_b$ along $\hat{z}_s$ and $\hat{z}_b$ along $\hat{x}_s$. This gives a rotation matrix of $R_{sb} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. The translation has three components: along $\hat{x}_s$ is 1003.87, along $\hat{y}_s$ is 160.3 and along $\hat{z}_s$ is 317 giving a translation of $t_{sb} = [1003.87, 160.3, 317]^T$.

$$M = \begin{bmatrix} 0 & 0 & 1 & 1003.87 \\ 1 & 0 & 0 & 160.3 \\ 0 & 1 & 0 & 317 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c.

For space/base form screw axes, note the axes have been marked J1 to J7 on the diagram,

The direction of screw axes: J1 is along $\hat{z}_s$ giving $\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, J2, 6 are along $\hat{y}_s$ giving $\omega_2 = \omega_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, J3, 5, 7 are along $\hat{x}_s$ giving $\omega_3 = \omega_5 = \omega_7 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, J4 is in $-\hat{y}_s$ direction $\omega_4 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$.

When rotating around each of the axes:

J1: no linear motion on the origin of {s} giving $v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

J2: linear motion on the origin of {s} in $-\hat{x}_s$ direction at a distance of 317, and in $\hat{z}_s$ direction at a distance of 83.87 giving $v_2 = \begin{bmatrix} -317 \\ 0 \\ 83.87 \end{bmatrix}$.

J3: linear motion on the origin of {s} in $\hat{y}_s$ direction at a distance of 317, and in $-\hat{z}_s$ direction at a distance of 192.5 giving $v_3 = \begin{bmatrix} 0 \\ 317 \\ -192.5 \end{bmatrix}$.

J4: linear motion on the origin of {s} in $\hat{x}_s$ direction at a distance of 317, and in $-\hat{z}_s$ direction at a distance of 83.37+400=483.37 giving $v_4 = \begin{bmatrix} 317 \\ 0 \\ -483.37 \end{bmatrix}$.

J5: linear motion on the origin of {s} in $\hat{y}_s$ direction at a distance of 317, and in $-\hat{z}_s$ direction at a distance of $192.5 - 168.5 = 24$ giving $v_5 = \begin{bmatrix} 0 \\ 317 \\ -24 \end{bmatrix}$.

J6: linear motion on the origin of {s} in $-\hat{x}_s$ direction at a distance of 317, and in $\hat{z}_s$ direction at a distance of 83.37+400+400=883.37 giving $v_6 = \begin{bmatrix} -317 \\ 0 \\ 883.37 \end{bmatrix}$.

J7: linear motion on the origin of {s} in $\hat{y}_s$ direction at a distance of 317, and in $-\hat{z}_s$ direction at a distance of $192.5 - 32.2 = 160.3$ giving $v_5 = \begin{bmatrix} 0 \\ 317 \\ -160.3 \end{bmatrix}$.

Putting all together:

$$\mathcal{S}_{\text{list}} = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -317 & 0 & 317 & 0 & -317 & 0 \\ 0 & 0 & 317 & 0 & 317 & 0 & 317 \\ 0 & 83.87 & -192.5 & -483.87 & -24 & 883.87 & -160.3 \end{bmatrix}$$

For body form screw axes, note the reference to {b} instead of {s},

The direction of screw axes: J1 is along $\hat{y}_b$ giving $\omega_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, J2, 6 are along $\hat{x}_b$ giving $\omega_2 = \omega_6 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, J3, 5, 7 are along $\hat{z}_b$ giving $\omega_3 = \omega_5 = \omega_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, J4 is in $-\hat{x}_b$ direction $\omega_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$.

When rotating around each of the axes:

J7: no linear motion on the origin of {b} giving $v_7 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

J6: linear motion on the origin of {b} in $-\hat{y}_b$ direction at a distance of 120 giving $v_2 = \begin{bmatrix} 0 \\ -120 \\ 0 \end{bmatrix}$.

J5: linear motion on the origin of {b} in $\hat{y}_b$ direction at a distance of 136.3 giving $v_5 = \begin{bmatrix} 0 \\ 136.3 \\ 0 \end{bmatrix}$.

J4: linear motion on the origin of {b} in $\hat{y}_b$ direction at a distance of 400+120=520 giving $v_4 = \begin{bmatrix} 0 \\ 520 \\ 0 \end{bmatrix}$.

J3: linear motion on the origin of {b} in $-\hat{y}_b$ direction at a distance of 32.2 giving $v_3 = \begin{bmatrix} 0 \\ -32.2 \\ 0 \end{bmatrix}$.

J2: linear motion on the origin of {b} in $-\hat{y}_b$ direction at a distance of 400+400+120=920 giving $v_2 = \begin{bmatrix} 0 \\ -920 \\ 0 \end{bmatrix}$.

J1: linear motion on the origin of {s} in $-\hat{z}_b$ direction at a distance of $192.5 - 32.2 = 160.3$, and in $\hat{x}_b$ direction at a distance of $83.87 + 400 + 400 + 120 = 1023.87$ giving $v_1 = \begin{bmatrix} 1003.87 \\ 0 \\ -160.3 \end{bmatrix}$.

Putting all together:

$$\mathcal{B}_{list} = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1003.87 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -920 & -32.2 & 520 & 136.3 & -120 & 0 \\ -160.3 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

d.

The FK in PoE is given as

$$T_{sb}(\theta) = e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} e^{[\mathcal{S}_3]\theta_3} e^{[\mathcal{S}_4]\theta_4} e^{[\mathcal{S}_5]\theta_5} e^{[\mathcal{S}_6]\theta_6} e^{[\mathcal{S}_7]\theta_7} M$$

For $\theta = \left(0, \frac{\pi}{2}, 0, \frac{\pi}{2}, 0, \frac{\pi}{2}, 0\right)$

$$T_{sb}(\theta) = e^{[\mathcal{S}_2]\theta_2} e^{[\mathcal{S}_4]\theta_4} e^{[\mathcal{S}_6]\theta_6} M$$

$$= e^{[\mathcal{S}_2]\frac{\pi}{2}} e^{[\mathcal{S}_4]\frac{\pi}{2}} e^{[\mathcal{S}_6]\frac{\pi}{2}} M$$

Where

$$[\mathcal{S}_2] = \begin{bmatrix} [\omega_2] & v_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -317 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 83.87 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[\mathcal{S}_4] = \begin{bmatrix} [\omega_4] & v_4 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 317 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -483.87 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[\mathcal{S}_6] = \begin{bmatrix} [\omega_6] & v_6 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -317 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 883.87 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Note } [\omega] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

We need to solve for

$$T_{sb}(\theta) = e^{\begin{bmatrix} 0 & 0 & 1 & -317 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 83.87 \\ 0 & 0 & 0 & 0 \end{bmatrix}\frac{\pi}{2}} e^{\begin{bmatrix} 0 & 0 & -1 & 317 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -483.87 \\ 0 & 0 & 0 & 0 \end{bmatrix}\frac{\pi}{2}} e^{\begin{bmatrix} 0 & 0 & 1 & -317 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 883.87 \\ 0 & 0 & 0 & 0 \end{bmatrix}\frac{\pi}{2}} \begin{bmatrix} 0 & 0 & 1 & 1003.87 \\ 1 & 0 & 0 & 160.3 \\ 0 & 1 & 0 & 317 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= e^{\begin{bmatrix} 0 & 0 & \frac{\pi}{2} & -317\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \\ -\frac{\pi}{2} & 0 & 0 & 83.87\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}} e^{\begin{bmatrix} 0 & 0 & -\frac{\pi}{2} & 317\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \\ \frac{\pi}{2} & 0 & 0 & -483.87\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}} e^{\begin{bmatrix} 0 & 0 & \frac{\pi}{2} & -317\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \\ -\frac{\pi}{2} & 0 & 0 & 883.87\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}} \begin{bmatrix} 0 & 0 & 1 & 1003.87 \\ 1 & 0 & 0 & 160.3 \\ 0 & 1 & 0 & 317 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where

$$e^{\begin{bmatrix} 0 & 0 & \frac{\pi}{2} & -317\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \\ -\frac{\pi}{2} & 0 & 0 & 83.87\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} 0 & 0 & 1 & -233.1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 400.87 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{\begin{bmatrix} 0 & 0 & -\frac{\pi}{2} & 317\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \\ \frac{\pi}{2} & 0 & 0 & -483.87\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} 0 & 0 & -1 & 800.87 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -166.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{\begin{bmatrix} 0 & 0 & \frac{\pi}{2} & -317\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \\ -\frac{\pi}{2} & 0 & 0 & 883.87\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} 0 & 0 & 1 & 566.87 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1200.87 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

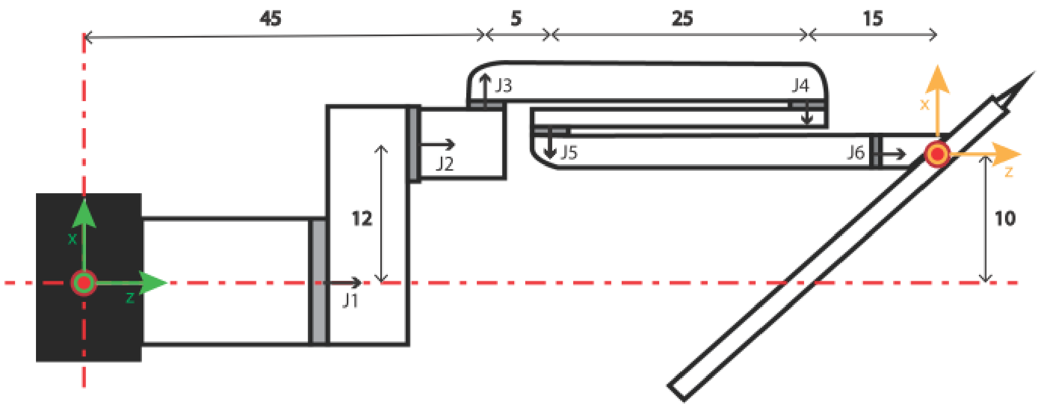
Note $e^{[\hat{\omega}]\theta} = I + \sin\theta\,[\hat{\omega}] + (1-\cos\theta)[\hat{\omega}]^2$. This calculation is best performed using computer.

Giving

$$T_{sb}(\theta) = \begin{bmatrix} 0 & 0 & 1 & -233.1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 400.87 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 800.87 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -166.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 566.87 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1200.87 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1003.87 \\ 1 & 0 & 0 & 160.3 \\ 0 & 1 & 0 & 317 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 483.87 \\ 1 & 0 & 0 & 160.3 \\ 0 & 0 & -1 & -203 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 4: da Vinci Xi manipulator arm



a.

We can see that {b} at home configuration as shown in the figure has the same orientation with {s}. This gives a rotation matrix of identity matrix, $R_{sb} = I$. The translation is at a distance of $45 + 5 + 25 + 15 = 90\ cm$ in the direction of $\hat{z}_s$ and $10\ cm$ in the $\hat{x}_s$ direction giving a translation of $t_{sb} = [10,0,90]^T$.

$$M = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 90 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b.

For space/base form screw axes, note the axes have been marked J1 to J6 on the diagram,

The direction of screw axes: J1, 2, 6 are along $\hat{z}_s$ giving $\omega_1 = \omega_2 = \omega_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, J3 is along $\hat{x}_s$ giving $\omega_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, J4, 5 are along $-\hat{x}_s$ giving $\omega_4 = \omega_5 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$.

When rotating around each of the axes:

J1: no linear motion on the origin of {s} giving $v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

J2: linear motion on the origin of {s} in $-\hat{y}_s$ direction at a distance of 12 giving $v_2 = \begin{bmatrix} 0 \\ -12 \\ 0 \end{bmatrix}$.

J3: linear motion on the origin of {s} in $\hat{y}_s$ direction at a distance of 45 giving $v_3 = \begin{bmatrix} 0 \\ 45 \\ 0 \end{bmatrix}$.

J4: linear motion on the origin of {s} in $-\hat{y}_s$ direction at a distance of 45+5+25=75 giving $v_4 = \begin{bmatrix} 0 \\ -75 \\ 0 \end{bmatrix}$.

J5: linear motion on the origin of {s} in $-\hat{y}_s$ direction at a distance of $45 + 5 = 50$ giving $v_5 = \begin{bmatrix} 0 \\ -50 \\ 0 \end{bmatrix}$.

J6: linear motion on the origin of {s} in $-\hat{y}_s$ direction at a distance of 10 giving $v_6 = \begin{bmatrix} 0 \\ -10 \\ 0 \end{bmatrix}$.

Putting all together:

$$\mathcal{S}_{\text{list}} = \begin{bmatrix} 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -12 & 45 & -75 & -50 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

c.

The FK in PoE is given as

$$T_{sb}(\theta) = e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} e^{[\mathcal{S}_3]\theta_3} e^{[\mathcal{S}_4]\theta_4} e^{[\mathcal{S}_5]\theta_5} e^{[\mathcal{S}_6]\theta_6} M$$

For $\theta = \left(0, \frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{2}\right)$

$$T_{sb}(\theta) = e^{[\mathcal{S}_2]\theta_2} e^{[\mathcal{S}_4]\theta_4} e^{[\mathcal{S}_5]\theta_5} e^{[\mathcal{S}_6]\theta_6} M$$

$$= e^{[\mathcal{S}_2]\frac{\pi}{4}} e^{[\mathcal{S}_4]\frac{\pi}{4}} e^{[\mathcal{S}_5]\frac{3\pi}{4}} e^{[\mathcal{S}_6]\frac{\pi}{2}} M$$

Where

$$[\mathcal{S}_2] = \begin{bmatrix} [\omega_2] & v_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[\mathcal{S}_4] = \begin{bmatrix} [\omega_4] & v_4 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -75 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[\mathcal{S}_5] = \begin{bmatrix} [\omega_5] & v_5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -50 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[\mathcal{S}_6] = \begin{bmatrix} [\omega_6] & v_6 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Note } [\omega] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

We need to solve for

$$T_{sb}(\theta) = e^{\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\frac{\pi}{4}} e^{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -75 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\frac{\pi}{4}} e^{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -50 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\frac{3\pi}{4}} e^{\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\frac{\pi}{2}} \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 90 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= e^{\begin{bmatrix} 0 & -\frac{\pi}{4} & 0 & 0 \\ \frac{\pi}{4} & 0 & 0 & -12\frac{\pi}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} e^{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\pi}{4} & -75\frac{\pi}{4} \\ 0 & -\frac{\pi}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} e^{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3\pi}{4} & -50\frac{3\pi}{4} \\ 0 & -\frac{3\pi}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} e^{\begin{bmatrix} 0 & -\frac{\pi}{2} & 0 & 0 \\ \frac{\pi}{2} & 0 & 0 & -10\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 90 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where

$$e^{\begin{bmatrix} 0 & -\frac{\pi}{4} & 0 & 0 \\ \frac{\pi}{4} & 0 & 0 & -12\frac{\pi}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} 0.7071 & -0.707 & 0 & 3.5147 \\ 0.7071 & 0.7071 & 0 & -8.485 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\pi}{4} & -75\frac{\pi}{4} \\ 0 & -\frac{\pi}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.7071 & 0.7071 & -53.03 \\ 0 & -0.707 & 0.7071 & 21.967 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3\pi}{4} & -50\frac{3\pi}{4} \\ 0 & -\frac{3\pi}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.707 & 0.7071 & -35.36 \\ 0 & -0.707 & -0.707 & 85.355 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{\begin{bmatrix} 0 & -\frac{\pi}{2} & 0 & 0 \\ \frac{\pi}{2} & 0 & 0 & -10\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} = \begin{bmatrix} 0 & -1 & 0 & 10 \\ 1 & 0 & 0 & -10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note $e^{[\hat{\omega}]\theta} = I + \sin\theta\,[\hat{\omega}] + (1-\cos\theta)[\hat{\omega}]^2$. This calculation is best performed using computer.

Giving

$$T_{sb}(\theta) = \begin{bmatrix} 0 & 0 & 1 & -233.1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 400.87 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 800.87 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -166.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 566.87 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1200.87 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1003.87 \\ 1 & 0 & 0 & 160.3 \\ 0 & 1 & 0 & 317 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7071 & 0.7071 & 0 & 23.086 \\ 0.7071 & -0.7071 & 0 & -13.91 \\ 0 & 0 & -1 & 17.322 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$