Tutorial 2: Rigid Body Motions

These questions are from the Practice Exercises of the Modern Robotics book. The solutions can be found on the book website. Please try your best before referring to the solutions. You should understand how to solve the problems.

Question 1: Transformation

Let the orientation of {b} relative to {a} be

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

And a point p be represented in {a} as $p_a = (1,2,3)$. What is p_b? Give a numeric 3-vector.

Question 2: Transformation

The mobile manipulator in Figure 1 needs to orient its gripper to grasp the block. For subsequent placement of the block, we have decided that the orientation of the gripper relative to the block, when the gripper grasps the block, should be R_{eg}. Our job is to determine the rotation operator to apply to the gripper to achieve this orientation relative to the block. Figure 1 shows the fixed world frame {a}, the mobile robot's chassis frame {b}, the gripper frame {c} (this is confusing, assume there is a fixed transformation from {c} to the actual gripper frame {g}, R_{cg}), the RGBD camera (color vision plus depth, like the Kinect) frame {d}, and the object frame {e}. Because we put the camera at a known location in space, we know R_{ad}. The camera reports the configuration of {e} relative to {d}, so we know R_{de}. From the mobile robot's localization procedure (e.g., vision-based localization or odometry) we know R_{ab}. From the robot arm's forward kinematics we know R_{bc}.

a. In terms of the four known rotation matrices R_{ad}, R_{de}, R_{ab}, and R_{bc}, and using only matrix multiplication and the transpose operation, express the current orientation of the gripper relative to the block, R_{ec}.
b. To align the gripper properly, you could apply to it a rotation $R1$ expressed in terms of axes in the gripper's {c} frame. What is $R1$, in terms of the five known rotation matrices (R_{ad}, R_{de}, R_{ab}, R_{bc}, R_{eg}), matrix multiplication, and transpose?

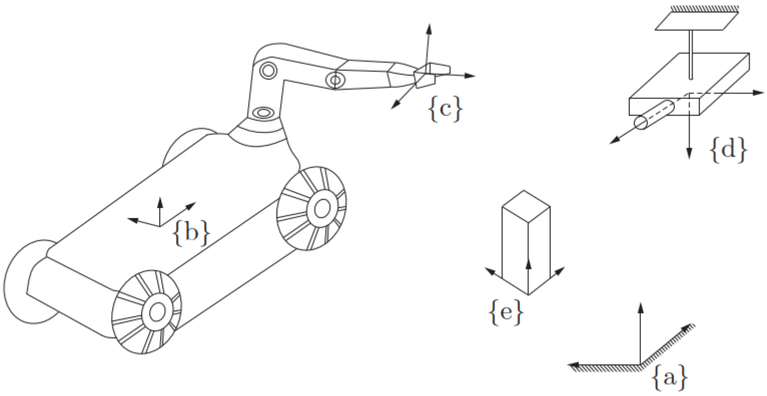

Figure 1 The fixed world frame {a}, the mobile robot's chassis frame {b}, the gripper frame {c}, the RGBD camera frame {d}, and the object frame {e}.

Question 3: Twist

Figure 2 shows a screw, a frame {b}, and a frame {s}. The $\hat{x}_b$ -axis of {b} is along the axis of the screw, and the origin of the frame {s} is displaced by 2 cm, along the $\hat{y}_b$ -axis, from the {b} frame. The $\hat{z}_s$ -axis is aligned with $\hat{x}_b$ and the $\hat{x}_s$ -axis is aligned with $\hat{z}_b$.

Taking note of the direction of the screw's threads, as the machine screw goes into a tapped hole driven by a screwdriver rotating at 3 radians per second with a pitch of 4π per revolution (this incur linear velocity), what is the screw's twist expressed in {b}, $\mathcal{V}_b$? What is the screw axis expressed in {b}, $\mathcal{S}_b$? What is $\mathcal{V}_s$? What is $\mathcal{S}_s$? Give units as appropriate.

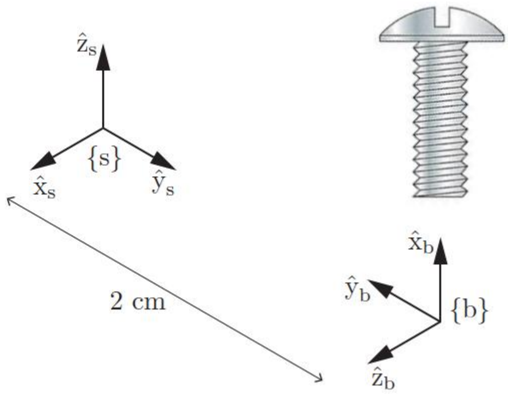

Figure 2 As the machine screw goes into a tapped hole, it advances linearly by 4π mm every full rotation of the screw.