Tutorial 1: Configuration space

These questions are from the Practice Exercises of the Modern Robotics book. The solutions can be found on the book website. Please try your best before referring to the solutions. You should understand how to solve the problems.

Question 1: Grubler's formula

The experimental surgical manipulator shown in *Figure 1*, developed at the National University of Singapore, is a parallel mechanism with three identical legs, each with a prismatic joint and two universal joints (the joints are marked for one of the legs). Use Grubler's formula to calculate the number of degrees of freedom of this mechanism.

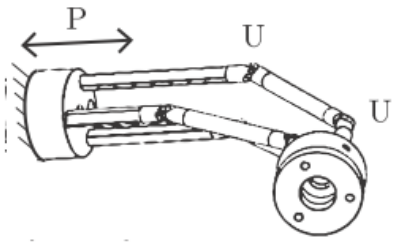

Figure 1. A miniature parallel surgical manipulator with three PUU legs.

Question 2: Degree of freedom

a. Three rigid bodies move in space independently. How many degrees of freedom does this system of three bodies have?
b. Now you constrain them so that each body must make contact with at least one of the other two bodies. (The bodies are allowed to slide and roll relative to each other, but they must remain in contact.) How many degrees of freedom does this system of three bodies have?

Question 3: C-space topology

A unicycle is controlled moving on a rigid balance beam as shown in *Figure 2*. Suppose the wheel is always touching the beam with no sliding, answer the following questions in terms of $\mathbb{R}, S, T$, and I (a one-dimensional closed interval).

a. Give a mathematical description of the C-space of the unicycle when it remains upright and is constrained to move in the 2-dimensional plane of the page.
b. Give a mathematical description of the C-space of the unicycle when it remains upright, it moves in a 3-dimensional space, and the beam has nonzero width.

Figure 2. A unicycle on a rigid balance beam.

Question 4: C-space representation

Imagine a C-space described as a circle in an (x, y) plane, of radius 2 centered at $(3,0)$. What is an implicit representation of this one-dimensional C-space? If you were to decide to parameterize the one-dimensional C-space by the single parameter θ, give a mapping from θ to (x, y).