Tutorial 1: Configuration space

These questions are from the Practice Exercises of the Modern Robotics book. The solutions can be found on the book website. Please try your best before referring to the solutions. You should understand how to solve the problems.

Question 1: Grubler’s formula

The experimental surgical manipulator shown in *Figure 1*, developed at the National University of Singapore, is a parallel mechanism with three identical legs, each with a prismatic joint and two universal joints (the joints are marked for one of the legs). Use Grubler’s formula to calculate the number of degrees of freedom of this mechanism.

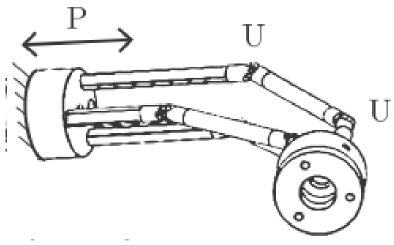


Figure 1. A miniature parallel surgical manipulator with three PUU legs.

Question 2: Degree of freedom

a. Three rigid bodies move in space independently. How many degrees of freedom does this system of three bodies have?
b. Now you constrain them so that each body must make contact with at least one of the other two bodies. (The bodies are allowed to slide and roll relative to each other, but they must remain in contact.) How many degrees of freedom does this system of three bodies have?

Question 3: C-space topology

A unicycle is controlled moving on a rigid balance beam as shown in *Figure 2*. Suppose the wheel is always touching the beam with no sliding, answer the following questions in terms of $\mathbb{R}, S, T$, and I (a one-dimensional closed interval).

a. Give a mathematical description of the C-space of the unicycle when it remains upright and is constrained to move in the 2-dimensional plane of the page.
b. Give a mathematical description of the C-space of the unicycle when it remains upright, it moves in a 3-dimensional space, and the beam has nonzero width.

Figure 2. A unicycle on a rigid balance beam.

Question 4: C-space representation

Imagine a C-space described as a circle in an (x, y) plane, of radius 2 centered at $(3,0)$. What is an implicit representation of this one-dimensional C-space? If you were to decide to parameterize the one-dimensional C-space by the single parameter θ, give a mapping from θ to (x, y).

Sample solution

Taken from Modern Robotics Practice Exercises, with additional notes.

Question 1: Grubler's formula

Solution 2.1 There are $N = 8$ links (two links in each leg, ground, and the moving platform). There are $J = 9$ joints (three prismatic joints and six universal joints). The joints have a total of $3(1) + 6(2) = 15$ degrees of freedom. By Grübler's formula,

$$\text{dof} = 6(8 - 1 - 9) + 15 = 3.$$

Question 2: Degree of freedom

Solution 2.2

(a) $3(6) = 18$.

(b) The system of three bodies is now subject to two equality constraints. For example, if the three bodies are called A, B, and C, the constraints could be written as the two equations dist(A,B) $= 0$ and dist(B,C) $= 0$. These two constraints subtract two degrees of freedom, so there are 16 degrees of freedom now.

Question 3: C-space topology

Solution 2.5

(a) I: the point of contact on the beam (which determines the angle of the wheel, since rolling is enforced). If we treat the allowed contact points on the beam as an open inerval, then the space is topologically equivalent to $\mathbb{R}$.

(b) $I^2 \times T^2$: intervals correspond to limited beam contact locations, S^1 for heading direction of wheel, and S^1 for the point of contact on the wheel.

Question 4: C-space representation

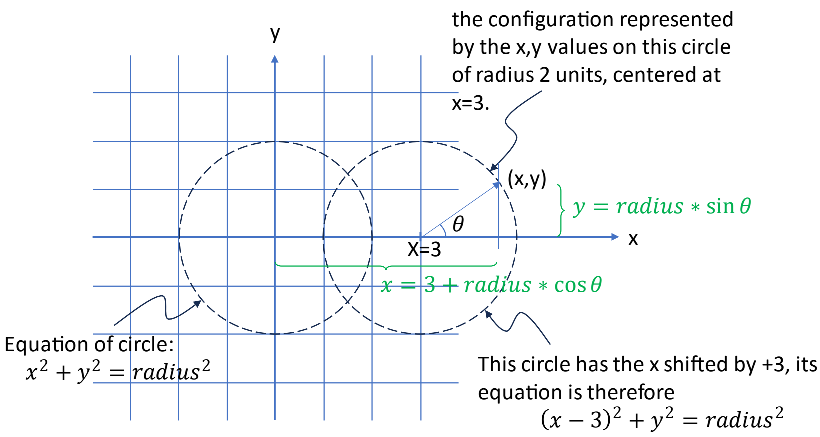
Note x,y cannot change independently, this system has 1 dof, i.e. if you change one of the variables, the other will change accordingly. Therefore (x,y) is an implicit representation. This representation has 2 dof (x and y) and is not complete without a constraint to ensure (x,y) falls on the circle.

Implicit representation is therefore: (x,y) with the constraint of $(x-3)^2 + y^2 = 2^2$.

At its minimal, we can simply represent the configuration with the angle of rotation of a vector of length 2 (radius), i.e. θ. This is an explicit representation: θ.

We can map θ to (x,y) by x and y components of the vector.

$$x = 3 + 2\cos\theta \text{ and } y = 2\sin\theta$$



Solution 2.8 Implicit: (x, y) such that $(x-3)^2 + y^2 = 4$. Explicit: $x = 3 + 2\cos\theta$, $y = 2\sin\theta$.