Velocity Kinematics

ZA-2203 Robotic Systems

Topics

- Velocity kinematics
- Jacobian
- Singularities
- Manipulability ellipsoid
- Spatial Jacobian
- Body Jacobian

Velocity Kinematics

- In previous lecture, we studied the motion of the robot manipulator in terms of **displacement** (position and orientation). Specifically, we learned to determine the displacement (transformation) of the end-effector given the motion at the joints (joint variables either rotation or linear). This is **forward kinematics**.
- In this lecture, we will study the motion of the robot manipulator in terms of **velocity** (linear and rotational). Specifically, we will learn to determine the twist of the end-effector given the velocities and positions at the joints. This is **velocity kinematics**.

Jacobian

- If the pose of the end-effector is given by $q \in \mathbb{R}^m$, e.g. $q = (q_1, \cdots, q_6) = \left(x, y, z, \phi_{yaw}, \phi_{pitch}, \phi_{roll}\right)$, the **forward kinematics** can be written as:

$$q = f(\theta)$$

- where $\theta = (\theta_1, \cdots, \theta_n) \in \mathbb{R}^n$ is the joint parameters.
- If the manipulator is moving, we can express the pose and joint parameters as functions of time:

$$q(t) = f\left(\theta(t)\right)$$

- The **time derivation** of the above equation will give the velocities of the end-effector pose $\dot{q}$ as a function of the joint velocities $\dot{\theta}$, i.e. **velocity kinematics**.

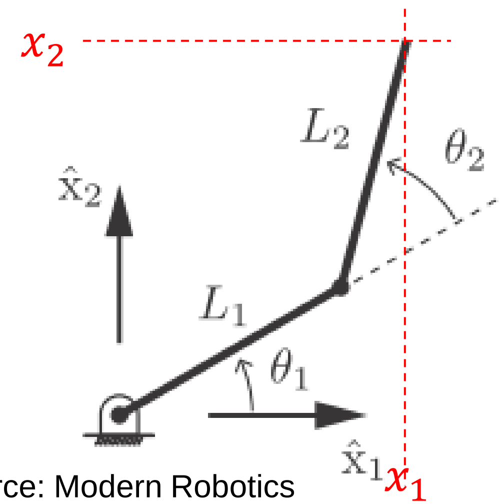
$$\dot{q} = \frac{dq(t)}{dt} = \frac{df\left(\theta(t)\right)}{dt} = \frac{\partial f(\theta)}{\partial \theta}\frac{d\theta(t)}{dt} = \frac{\partial f(\theta)}{\partial \theta}\dot{\theta} = J(\theta)\dot{\theta}$$

Jacobian

- $J(\theta) \in \mathbb{R}^{m \times n}$ is called the **Jacobian**. The Jacobian matrix represents the linear sensitivity of the end-effector velocity to the joint velocities $\dot{\theta}$.

$$\dot{q} = J(\theta)\dot{\theta}$$

(Jacobian maps joint velocity $\dot{\theta}$ to end-effector velocity $\dot{q}$)



Source: Modern Robotics

Forward kinematics (by geometry):

$$x_1 = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$
$$x_2 = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

Velocity kinematics (time derivative):

$$\dot{x}_1 = -L_1\dot{\theta}_1 \sin\theta_1 - L_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)\sin(\theta_1 + \theta_2)$$
$$\dot{x}_2 = L_1\dot{\theta}_1 \cos\theta_1 + L_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)\cos(\theta_1 + \theta_2)$$

$$q = (x_1, x_2),\ \theta = (\theta_1, \theta_2)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-L_1\sin\theta_1 - L_2\sin(\theta_1+\theta_2) & -L_2\sin(\theta_1+\theta_2)\\ L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) & L_2\cos(\theta_1+\theta_2)\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$

$J_1(\theta)$ $\quad$ $J_2(\theta)$

$$tip\ velocity\ v_{tip} = J_1(\theta)\dot{\theta}_1 + J_2(\theta)\dot{\theta}_2$$

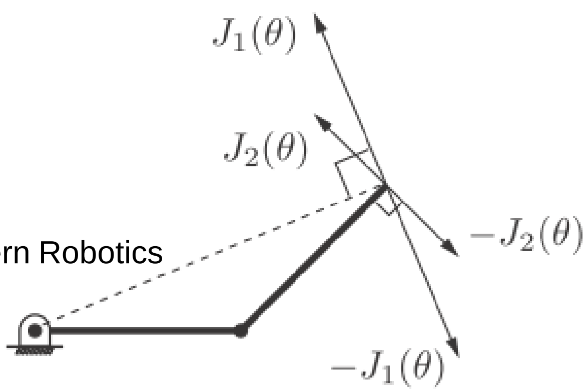
Jacobian

- Let $L_1 = L_2 = 1$, consider two nonsingular postures: $\theta = (0, {}^{\pi}/_{4})$ and $\theta = (0, {}^{3\pi}/_{4})$. The Jacobians $J(\theta)$ at these two configurations are

$$\begin{bmatrix} -L_1 \sin\theta_1 - L_2 \sin(\theta_1+\theta_2) & -L_2 \sin(\theta_1+\theta_2) \\ L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) & L_2 \cos(\theta_1+\theta_2) \end{bmatrix}$$

$$J\left(\begin{bmatrix} 0 \\ {}^{\pi}/_{4} \end{bmatrix}\right) = \begin{bmatrix} -0.71 & -0.71 \\ 1.71 & 0.71 \end{bmatrix} \quad \text{and} \quad J\left(\begin{bmatrix} 0 \\ {}^{3\pi}/_{4} \end{bmatrix}\right) = \begin{bmatrix} -0.71 & -0.71 \\ 0.29 & -0.71 \end{bmatrix}$$

$J_1(\theta)$ $J_2(\theta)$ $J_1(\theta)$ $J_2(\theta)$



Source: Modern Robotics

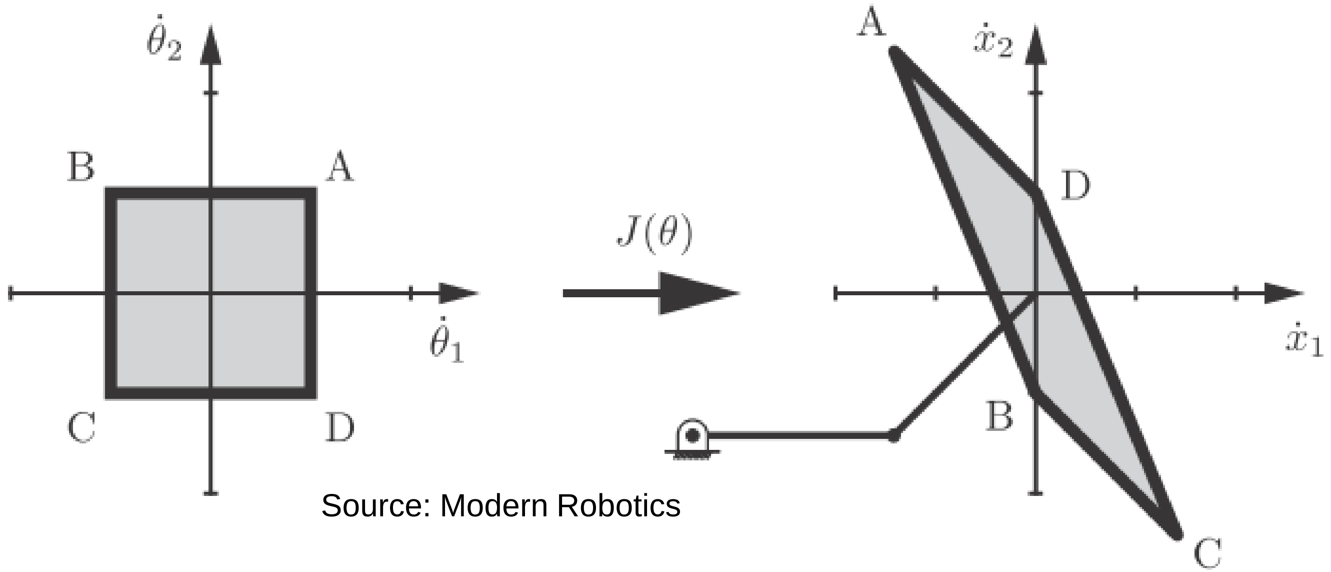
$\theta = (0, {}^{\pi}/_{4})$

$J_i(\theta)$ (each column) corresponds to the tip velocity when $\dot{\theta}_i = 1$ and the other joint velocity is zero

Singularities

- As long as $J_1(\theta)$ and $J_2(\theta)$ are not collinear (on the same straight line), it is possible to generate a tip velocity v_{tip} in any arbitrary direction in the x_1- x_2 -plane by choosing appropriate joint velocities.
- If a configuration of a manipulator results in $J_1(\theta)$ and $J_2(\theta)$ being **collinear**, then such configuration is called **singularity**.
- A **singularity** is characterized by a situation where the robot tip is unable to generate velocities in certain directions.

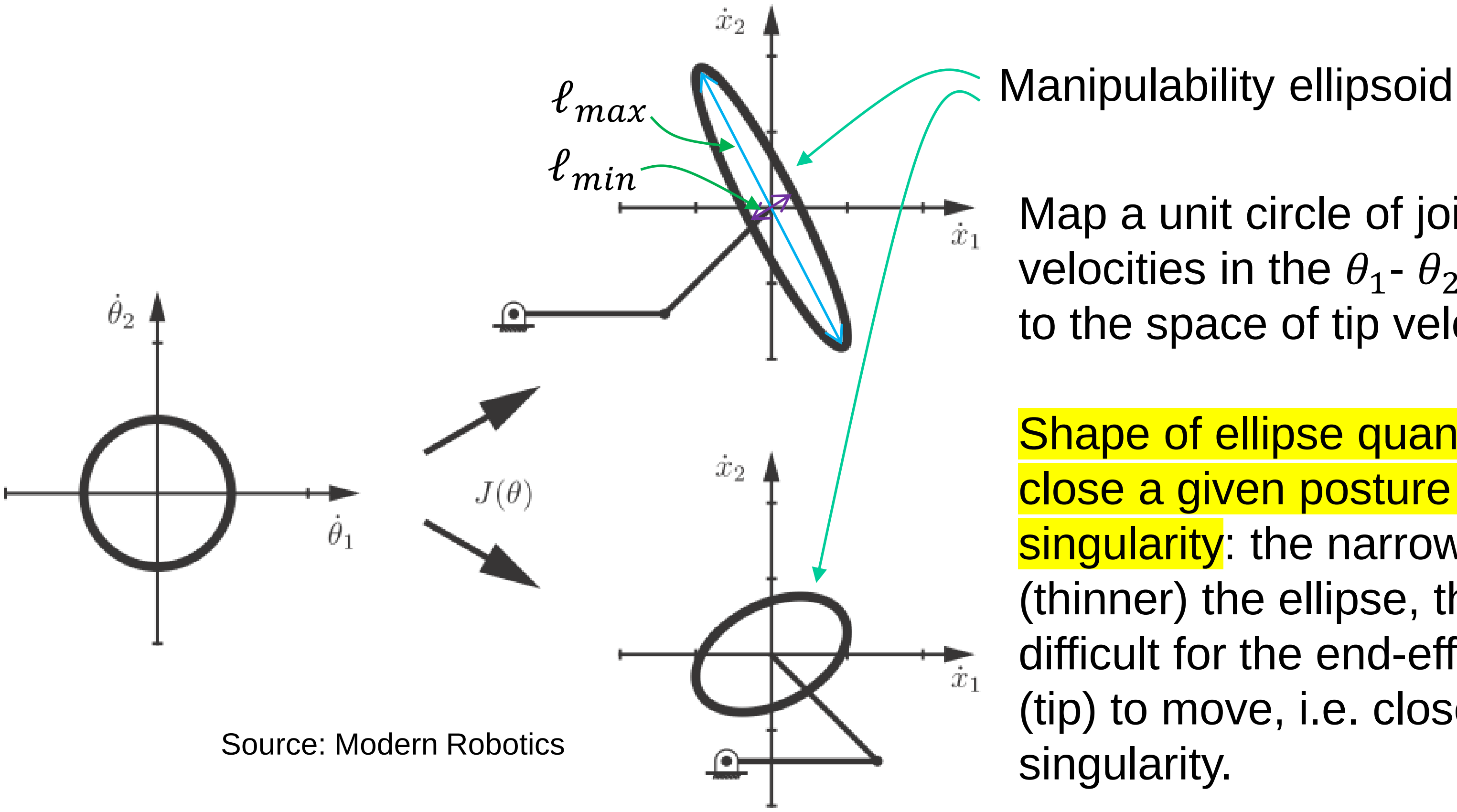
Map bounds on joint velocity to tip velocity



Source: Modern Robotics

The extreme points A, B, C, and D in the joint velocity space map to the extreme points A, B, C, and D in the end-effector velocity space.

Manipulability ellipsoid



Source: Modern Robotics

Map a unit circle of joint velocities in the θ_1- θ_2 -plane to the space of tip velocities.

Shape of ellipse quantity how close a given posture is to a singularity: the narrower (thinner) the ellipse, the more difficult for the end-effector (tip) to move, i.e. closer to singularity.

Ideally, $\frac{\ell_{min}}{\ell_{max}} = 1$. The closer to 1, the easier can the tip move in arbitrary directions.

Space Jacobian: by screw

- Let the forward kinematics of an n-link open chain be expressed in the following product of exponentials form:

$$T(\theta) = e^{[\mathcal{S}_1]\theta_1} \dots e^{[\mathcal{S}_n]\theta_n} M$$

- The space Jacobian $J_s(\theta) \in \mathbb{R}^{6\times n}$ relates the joint rate vector $\dot{\theta} \in \mathbb{R}^n$ to the spatial twist $\mathcal{V}_s$ via

$$\mathcal{V}_s = J_s(\theta)\dot{\theta}$$

- The ith column of $J_s(\theta)$ can be found by

$$J_{si}(\theta) = Ad_{e^{[\mathcal{S}_1]\theta_1}\dots e^{[\mathcal{S}_{i-1}]\theta_{i-1}}}(\mathcal{S}_i)$$

for $i = 2, \cdots, n$, with the first column $J_{s1} = \mathcal{S}_1$.

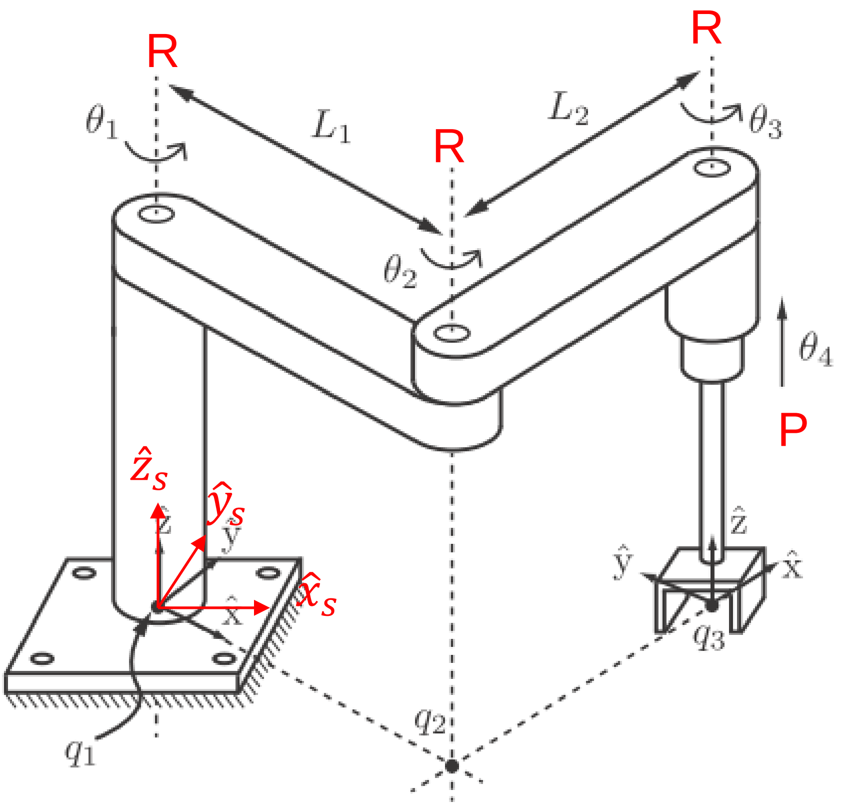
Space Jacobian: by screw

- To determine each column of the Jacobian $J_s(\theta)$
$$J_{si}(\theta) = Ad_{e^{[\mathcal{S}_1]\theta_1} \cdots e^{[\mathcal{S}_{i-1}]\theta_{i-1}}}(\mathcal{S}_i)$$
for $i = 2, \cdots, n$, with the first column $J_{s1} = \mathcal{S}_1$.
- Note $e^{[\mathcal{S}_1]\theta_1} \dots e^{[\mathcal{S}_{i-1}]\theta_{i-1}} = T_{i-1}$, and $Ad_{T_{i-1}}(\mathcal{S}_i)$ is therefore the screw axis describing the ith joint axis after it undergoes the rigid body displacement T_{i-1}.
- Physically this is the same as moving the first $i - 1$ joints from their zero position to the current values $\theta_1, \cdots, \theta_{i-1}$. Therefore, the i th column $J_{si}(\theta)$ of $J_s(\theta)$ is simply the screw vector describing joint axis i, expressed in fixed-frame coordinates, as a function of the joint variables $\theta_1, \cdots, \theta_{i-1}$.

Space Jacobian: by screw

- We can determine each column $J_{si}(\theta)$ similar to the procedure for deriving the joint screws in the product of exponentials $e^{[\mathcal{S}_1]\theta_1} \dots e^{[\mathcal{S}_n]\theta_n} M$: each column $J_{si}(\theta)$ is the screw vector describing joint axis i, expressed in fixed-frame coordinates, **but for arbitrary $\boldsymbol{\theta}$ rather than $\boldsymbol{\theta} = \mathbf{0}$**.

Space Jacobian E.g.1: Spatial RRRP chain



Source: Modern Robotics

Consider an RRRP robotic arm in 3D (spatial).

We see there are 4 joint variables, θ_1, θ_2, θ_3, θ_4. Therefore $n = 4$.

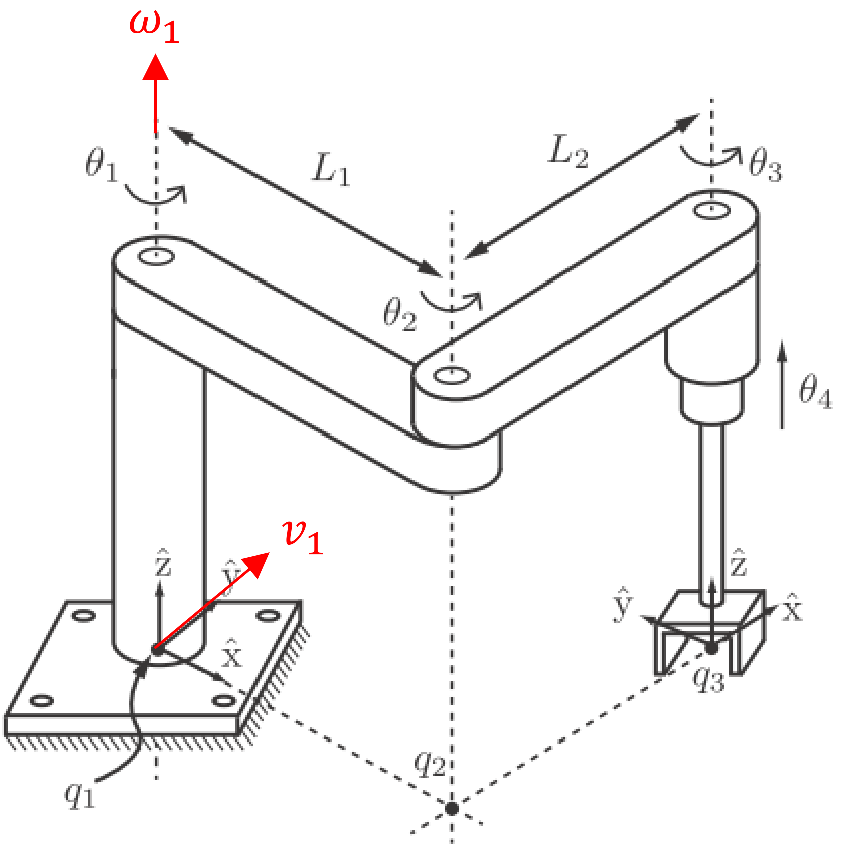
The Jacobian matrix therefore has 4 columns.

$$J_s(\theta) = [J_{s1}(\theta) \quad J_{s2}(\theta) \quad J_{s3}(\theta) \quad J_{s4}(\theta)]$$

Each column is the screw axis at each joint. E.g. $J_{s1}(\theta)$ is the screw axis at joint 1.

$$J_{s1}(\theta) = \begin{bmatrix} \omega_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} \omega_{x1} \\ \omega_{y1} \\ \omega_{z1} \\ v_{x1} \\ v_{y1} \\ v_{z1} \end{bmatrix}$$

Space Jacobian E.g.1: Spatial RRRP chain



Source: Modern Robotics

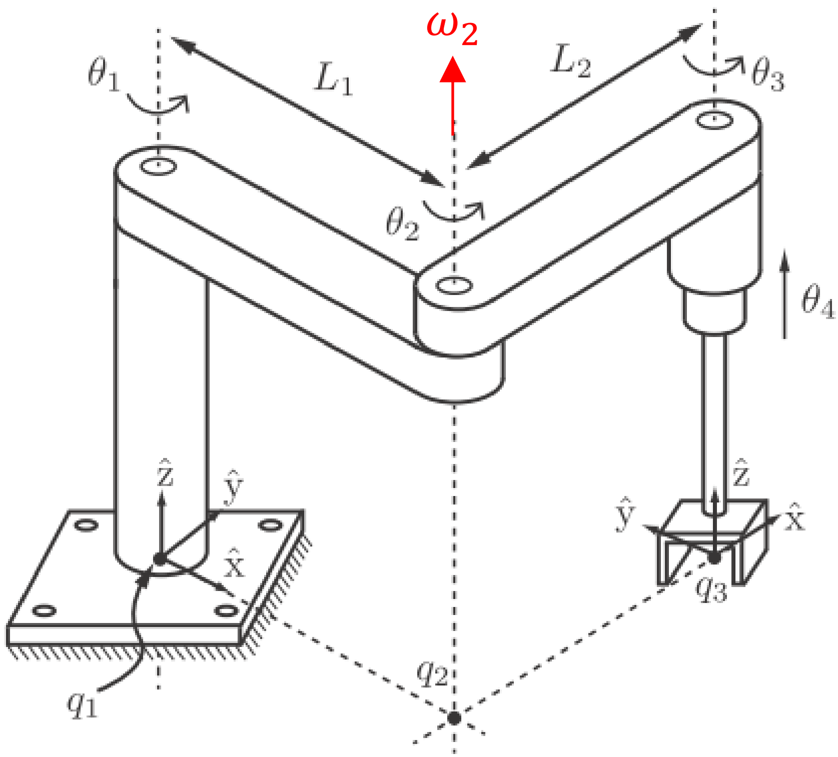
To determine $J_{s1}(\theta)$, we look at joint 1.

We see ω_1 points in z_s-direction.

v_1 is the linear velocity of a point at origin {s} when link 1 rotates around ω_1. There is no linear movement when rotating around ω_1.

$$J_{s1}(\theta) = \begin{bmatrix} \omega_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Space Jacobian E.g.1: Spatial RRRP chain



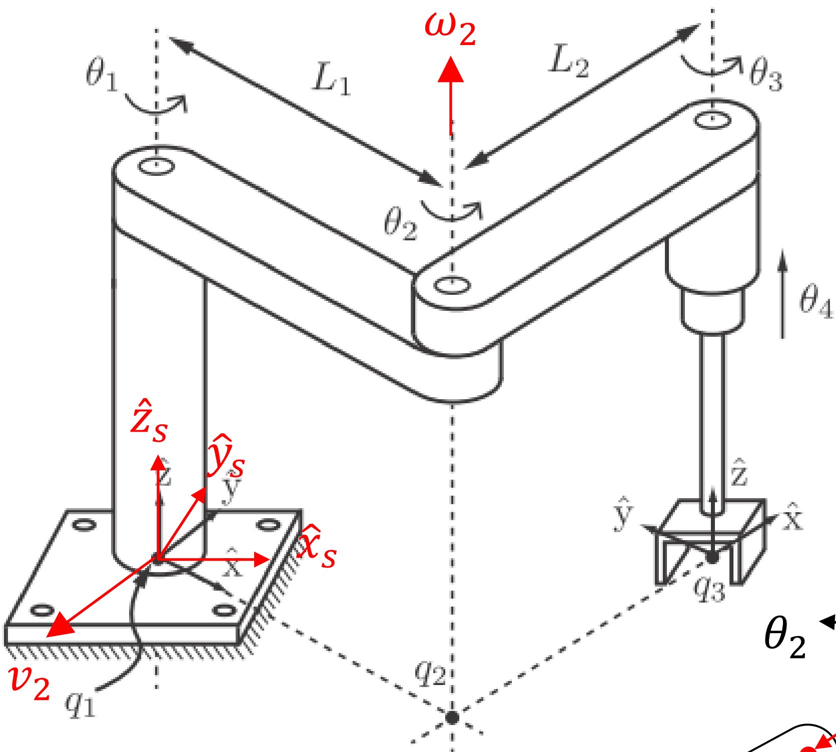
Source: Modern Robotics

To determine $J_{s2}(\theta)$, we look at joint 2.

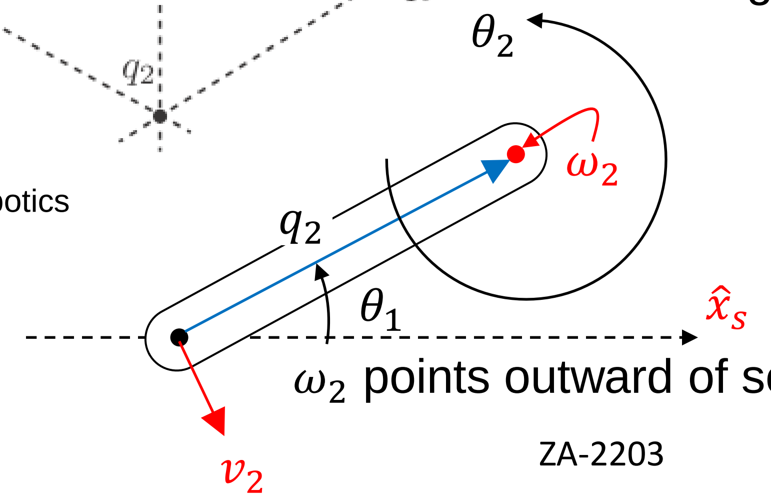
We see ω_2 points in z_s-direction. When considering ω_2 we need to take into account of the effect of θ_1, i.e. we want to define ω_2 for any arbitrary value of θ_1. However, we note the direction of ω_2 is not affected by joint 1 movement (θ_1).

$$\omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Space Jacobian E.g.1: Spatial RRRP chain



Source: Modern Robotics



ω_2 points outward of screen

v_2 is the linear velocity of a point at origin {s} when link 2 rotates around ω_2. When considering v_2 we need to take into account of the effect of θ_1, i.e. we want to define v_2 for any arbitrary value of θ_1. Base on RHR,

$$v_2 = -\omega_2 \times q_2$$

where

$$\omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } q_2 = \begin{bmatrix} L_1 \cos\theta_1 \\ L_1 \sin\theta_1 \\ 0 \end{bmatrix}$$

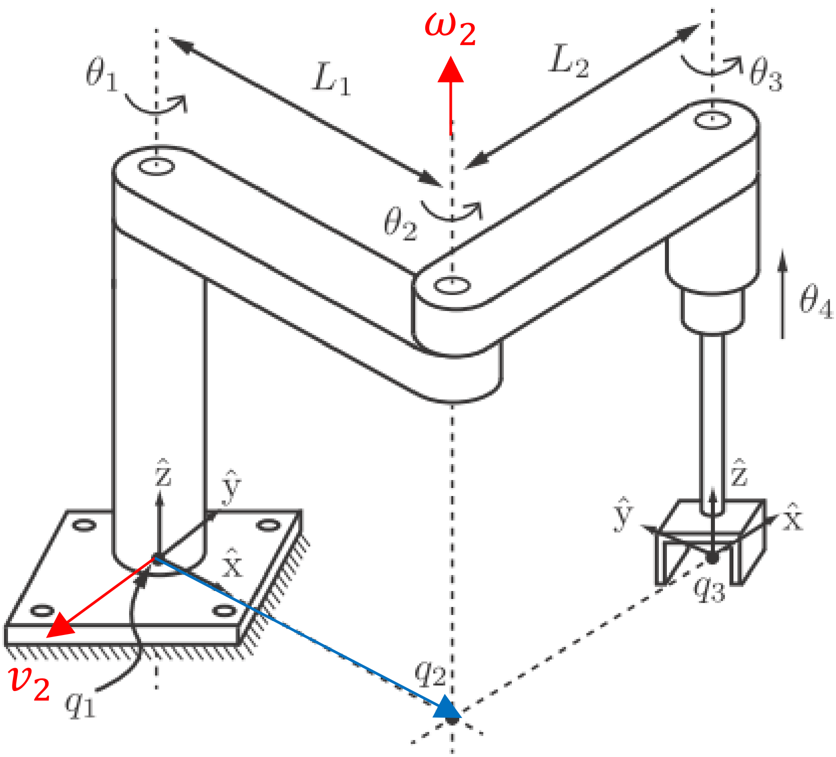
Giving

$$v_2 = \begin{bmatrix} L_1 \sin\theta_1 \\ -L_1 \cos\theta_1 \\ 0 \end{bmatrix}$$

Recall

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

Space Jacobian E.g.1: Spatial RRRP chain

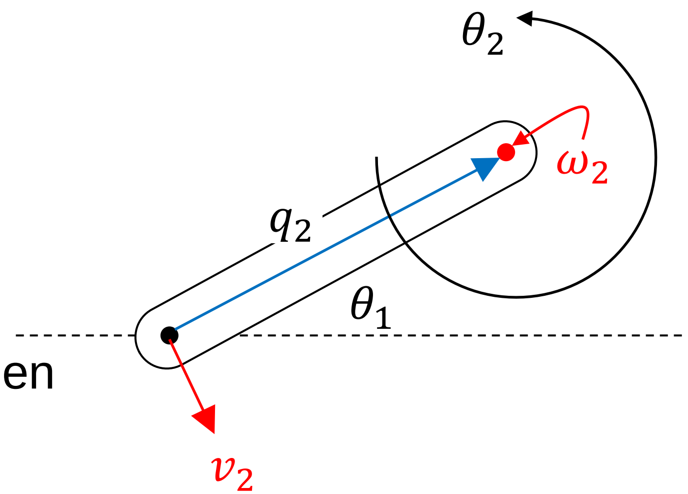


Source: Modern Robotics

To determine $J_{s2}(\theta)$, we look at joint 2.

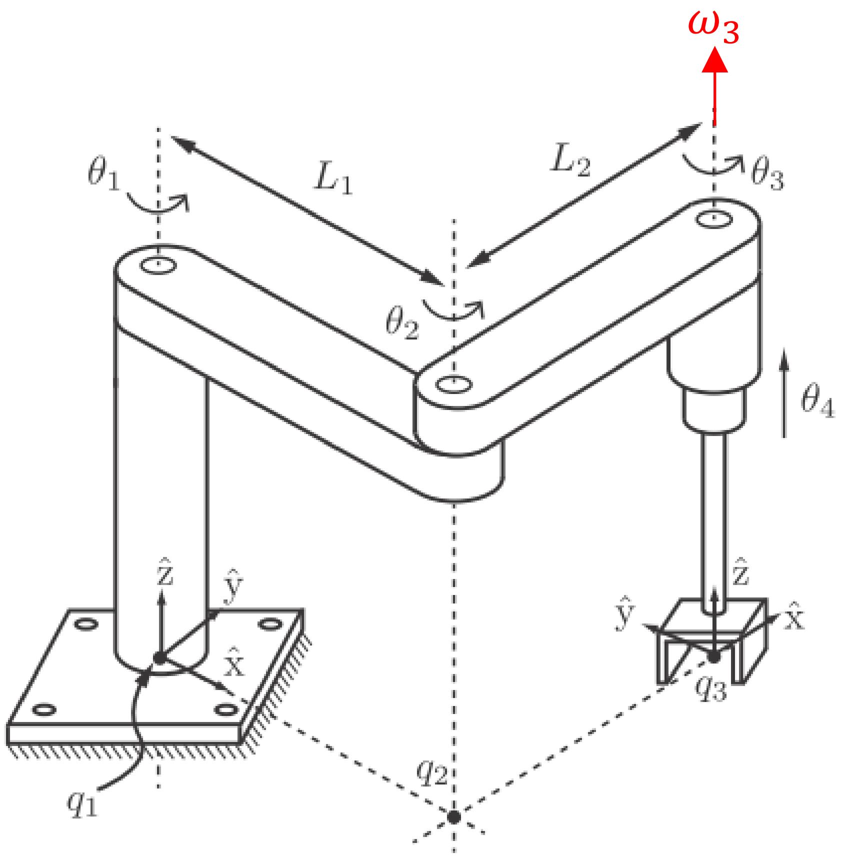
$$\omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } v_2 = \begin{bmatrix} L_1 \sin\theta_1 \\ -L_1 \cos\theta_1 \\ 0 \end{bmatrix}$$

$$J_{s2}(\theta) = \begin{bmatrix} \omega_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ L_1 s_1 \\ -L_1 c_1 \\ 0 \end{bmatrix}$$



ω_2 points outward of screen

Space Jacobian E.g.1: Spatial RRRP chain



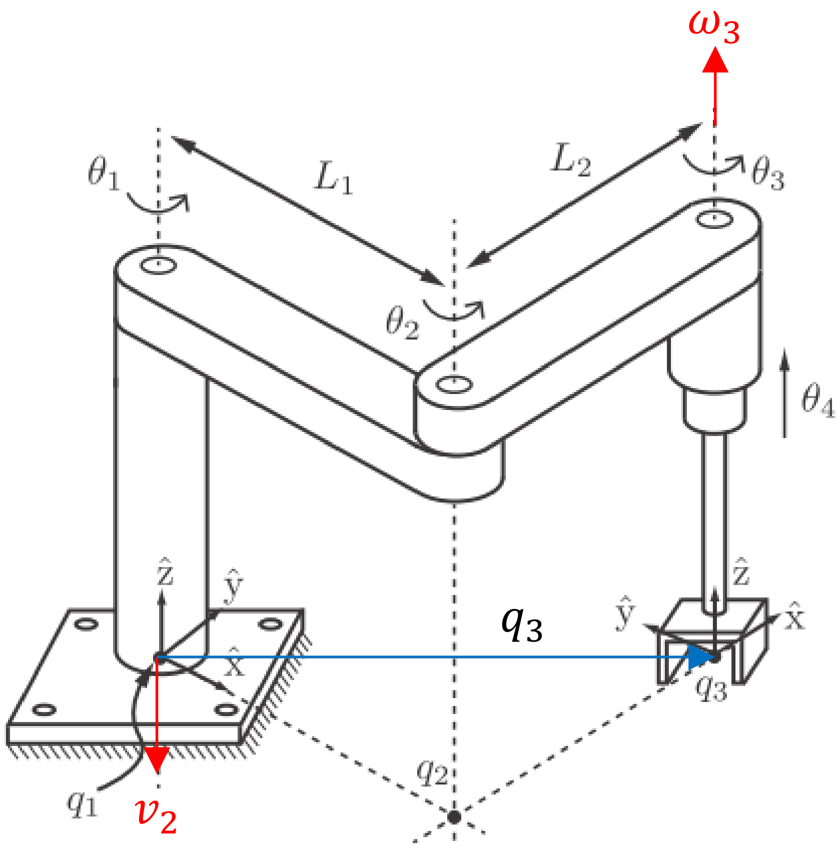
Source: Modern Robotics

To determine $J_{s3}(\theta)$, we look at joint 3. We need to consider the effect of θ_1 and θ_2 on ω_3 and v_3.

ω_3 is not affected by the movement of θ_1 and θ_2.

$$\omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Space Jacobian E.g.1: Spatial RRRP chain



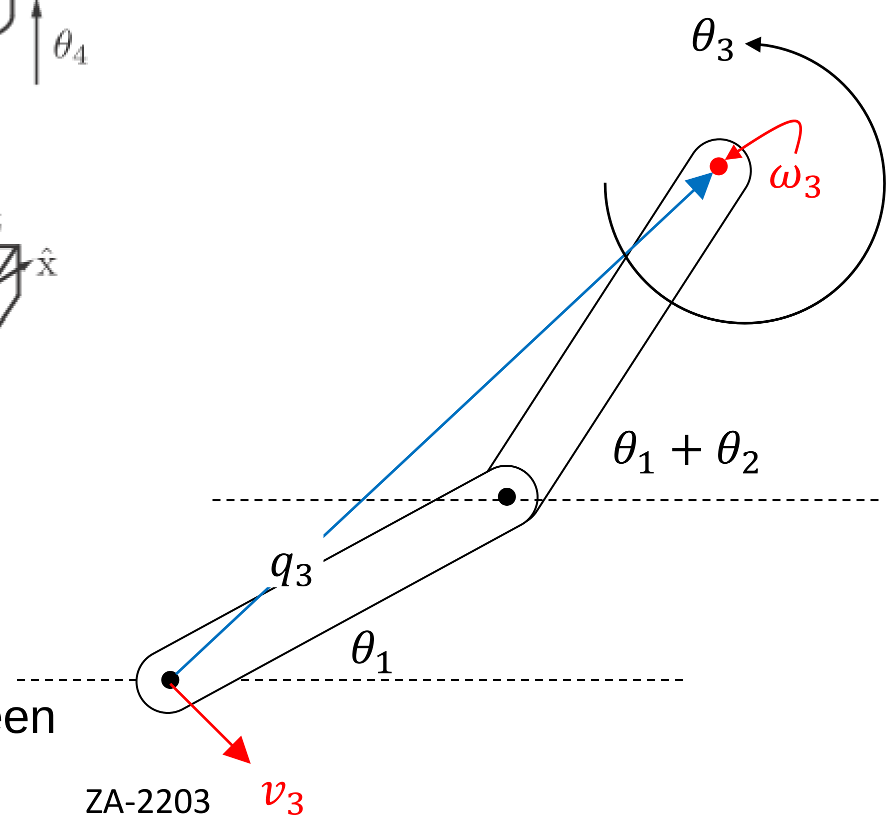
Source: Modern Robotics

Base on RHR,

$$v_3 = -\omega_3 \times q_3$$

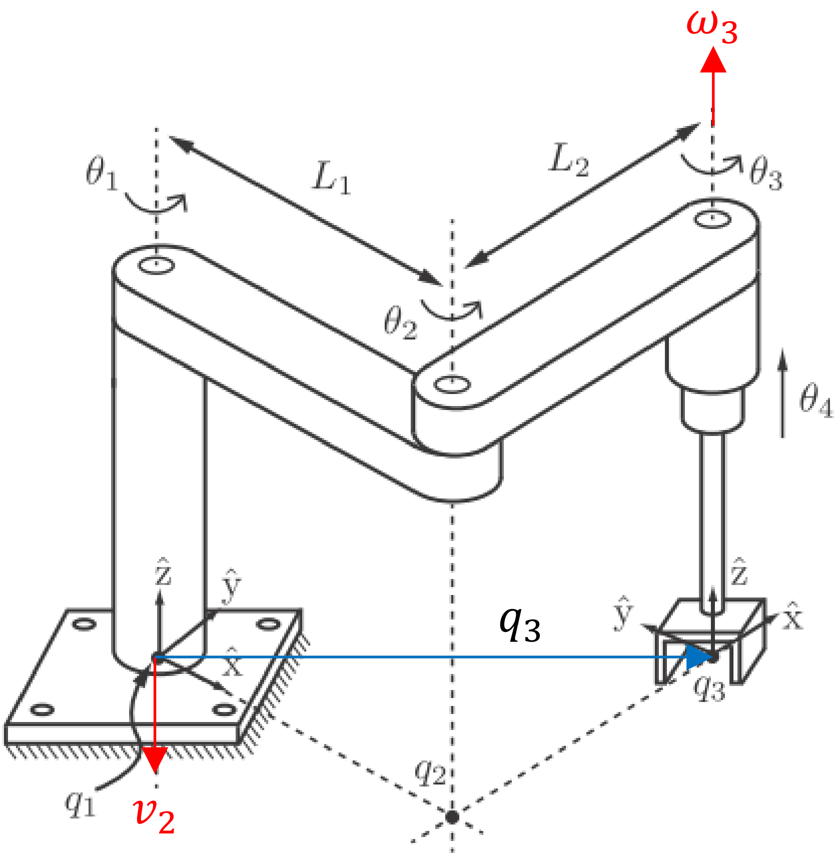
where

$$\omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } q_3 = \begin{bmatrix} L_1 c_1 + L_2 c_{12} \\ L_1 s_1 + L_2 s_{12} \\ 0 \end{bmatrix}$$



ω_3 points outward of screen

Space Jacobian E.g.1: Spatial RRRP chain



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Base on RHR,

$$v_3 = -\omega_3 \times q_3$$

where

$$\omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } q_3 = \begin{bmatrix} L_1 c_1 + L_2 c_{12} \\ L_1 s_1 + L_2 s_{12} \\ 0 \end{bmatrix}$$

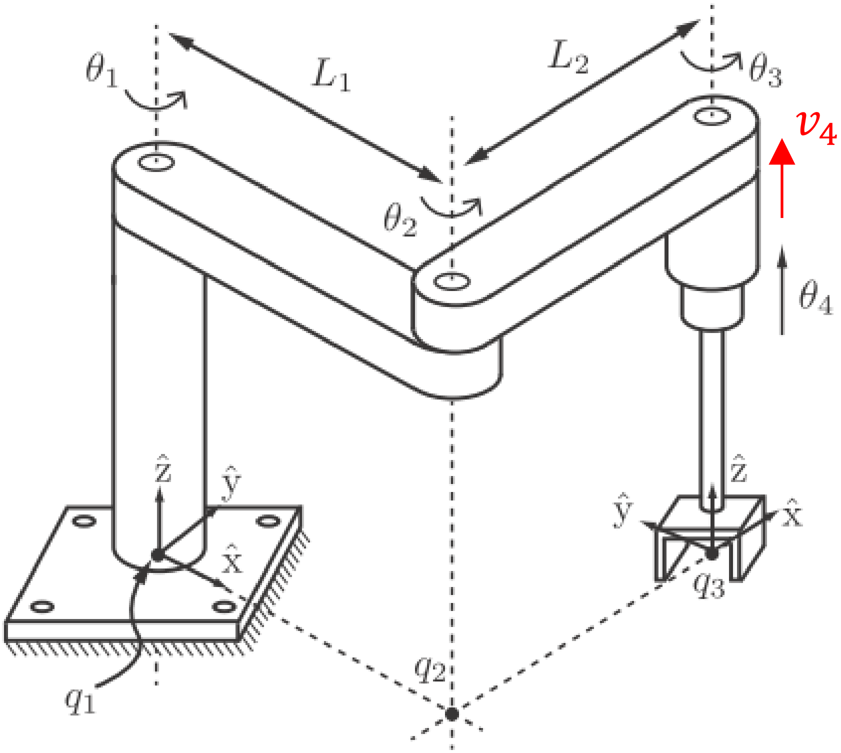
Giving $\quad v_3 = \begin{bmatrix} L_1 s_1 + L_2 s_{12} \\ -(L_1 c_1 + L_2 c_{12}) \\ 0 \end{bmatrix}$

$$J_{s3}(\theta) = \begin{bmatrix} \omega_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ L_1 s_1 + L_2 s_{12} \\ -(L_1 c_1 + L_2 c_{12}) \\ 0 \end{bmatrix}$$

Recall

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

Space Jacobian E.g.1: Spatial RRRP chain



Source: Modern Robotics

To determine $J_{s4}(\theta)$, we look at joint 4. We need to consider the effect of θ_1, θ_2 and θ_3 on ω_4 and v_4.

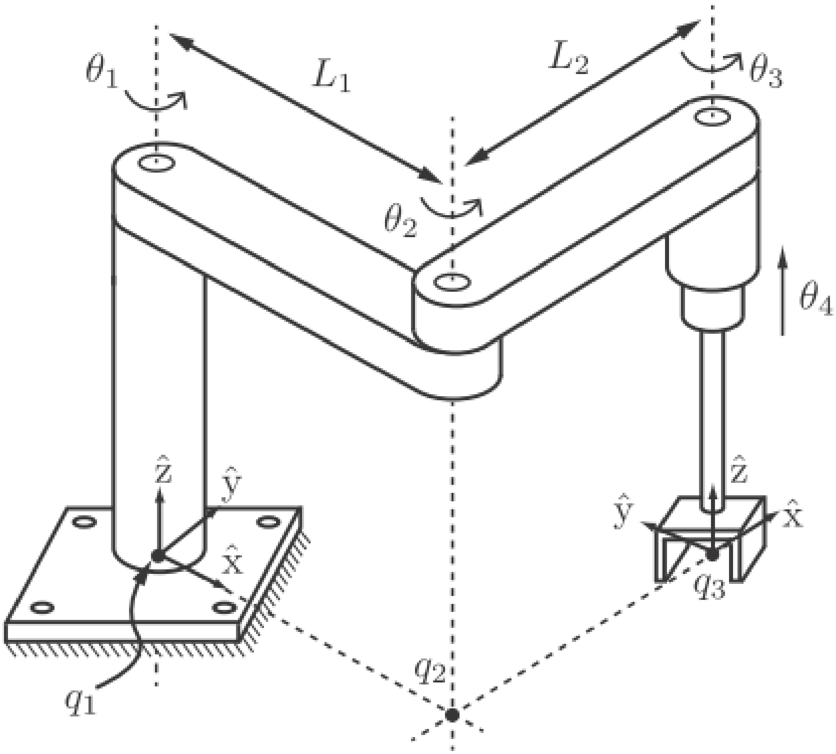
We take note that joint 4 is a prismatic joint, it has no angular velocity.

For prismatic joint, v_4 is the direction of the prismatic motion. In this case, it is in the direction of z_s-direction.

$$\omega_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } v_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_{s4}(\theta) = \begin{bmatrix} \omega_4 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

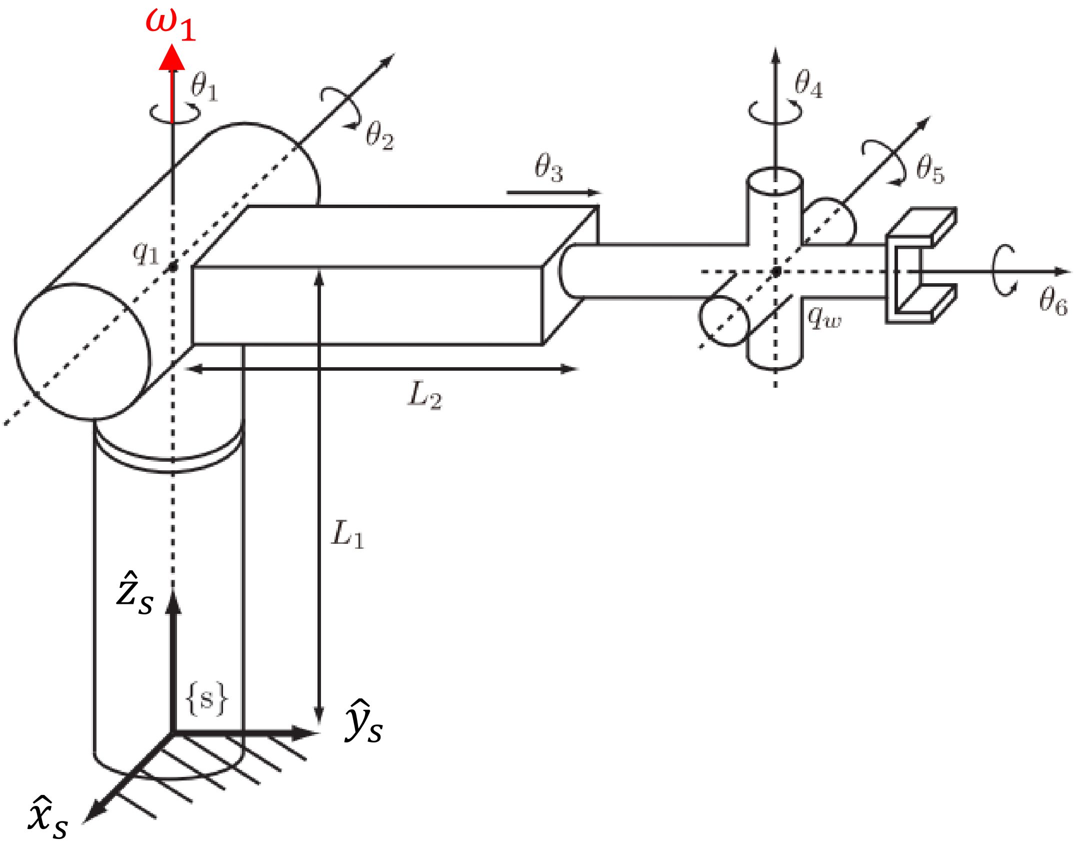
Space Jacobian E.g.1: Spatial RRRP chain



Source: Modern Robotics

$$J_s(\theta) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & L_1 s_1 & L_1 s_1 + L_2 s_{12} & 0 \\ 0 & -L_1 c_1 & -L_1 c_1 - L_2 c_{12} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Space Jacobian E.g.2: Spatial RRPRRR chain



Source: Modern Robotics

For $J_{s1}(\theta)$, we look at joint 1.

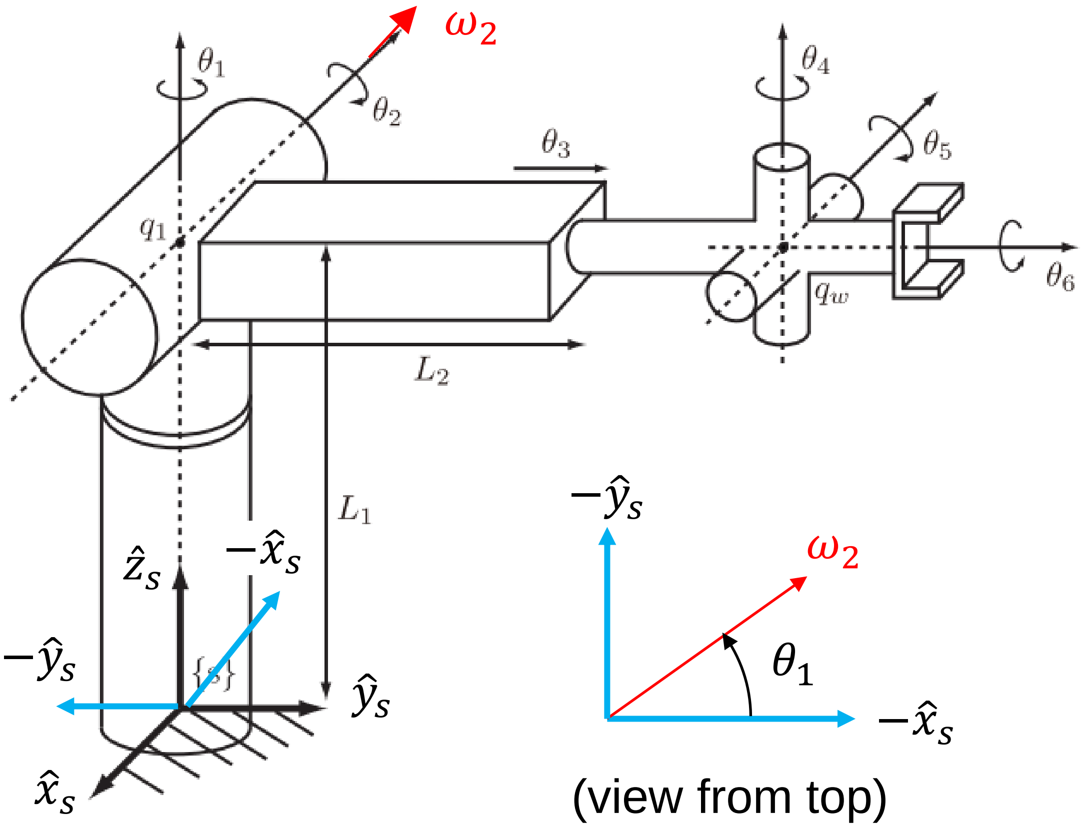
ω_1 is in the direction of z_s-direction.

There is no linear motion when rotating around this axis.

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ and } v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$J_{s1}(\theta) = \begin{bmatrix} \omega_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Space Jacobian E.g.2: Spatial RRPRRR chain



(view from top)

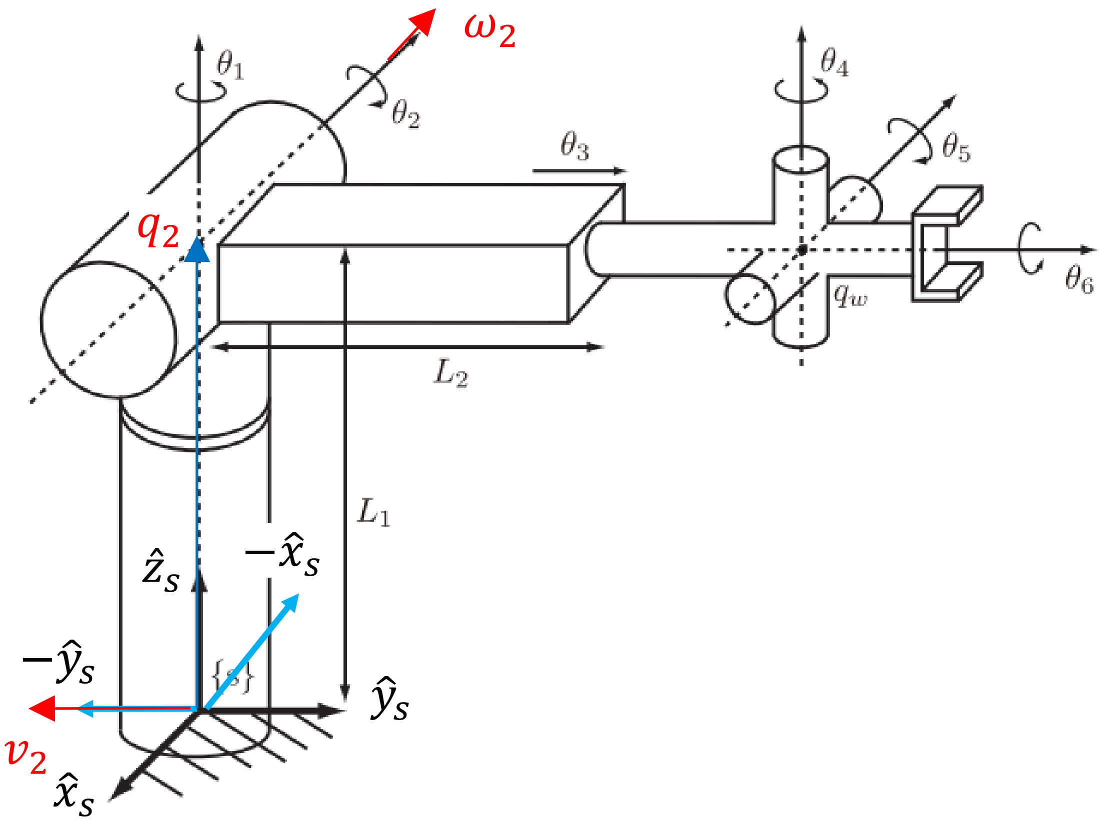
Source: Modern Robotics

For $J_{s2}(\theta)$, we look at joint 2 with consideration of change in θ_1.

ω_2 is on x_z-y_s plane, however the direction depends on θ_1. Take note of the zero configuration position of θ_1 to ensure we get the sign correct.

$$\omega_2 = \begin{bmatrix} -\cos\theta_1 \\ -\sin\theta_1 \\ 0 \end{bmatrix}$$

Space Jacobian E.g.2: Spatial RRPRRR chain



Source: Modern Robotics

For $J_{s2}(\theta)$, we look at joint 2 with consideration of change in θ_1.

$$v_2 = -\omega_2 \times q_2$$

$$\omega_2 = \begin{bmatrix} -\cos\theta_1 \\ -\sin\theta_1 \\ 0 \end{bmatrix}, q_2 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}$$

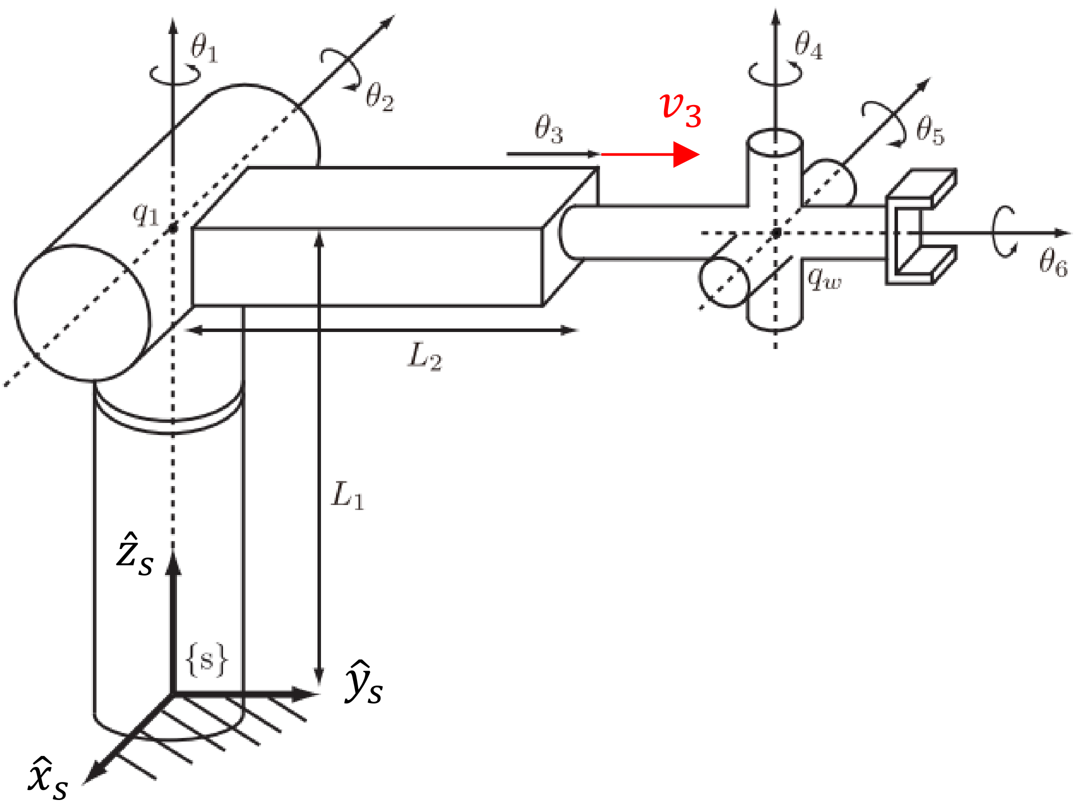
$$v_2 = \begin{bmatrix} L_1 s_1 \\ -L_1 c_1 \\ 0 \end{bmatrix}$$

$$J_{s2}(\theta) = \begin{bmatrix} \omega_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} -c_1 \\ -s_1 \\ 0 \\ L_1 s_1 \\ -L_1 c_1 \\ 0 \end{bmatrix}$$

Recall

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

Space Jacobian E.g.2: Spatial RRPRRR chain



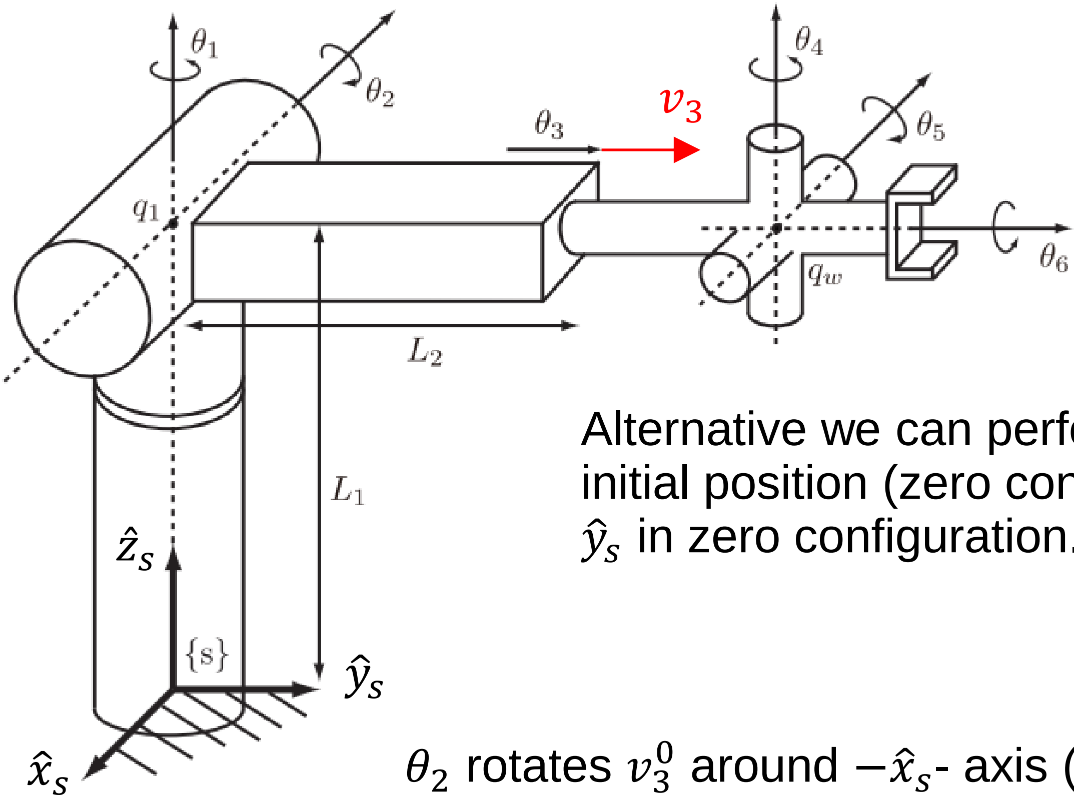
Source: Modern Robotics

For $J_{s3}(\theta)$, we look at joint 3 with consideration of changes in θ_1, θ_2.

Joint 3 is prismatic, so there is no angular velocity.

$$\omega_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Space Jacobian E.g.2: Spatial RRPRRR chain



Source: Modern Robotics

v_3 defines the direction of the linear motion. It is affected by θ_1, θ_2.

We can draw in 3D to determine the coordinates of v_3 by geometry and cross product.

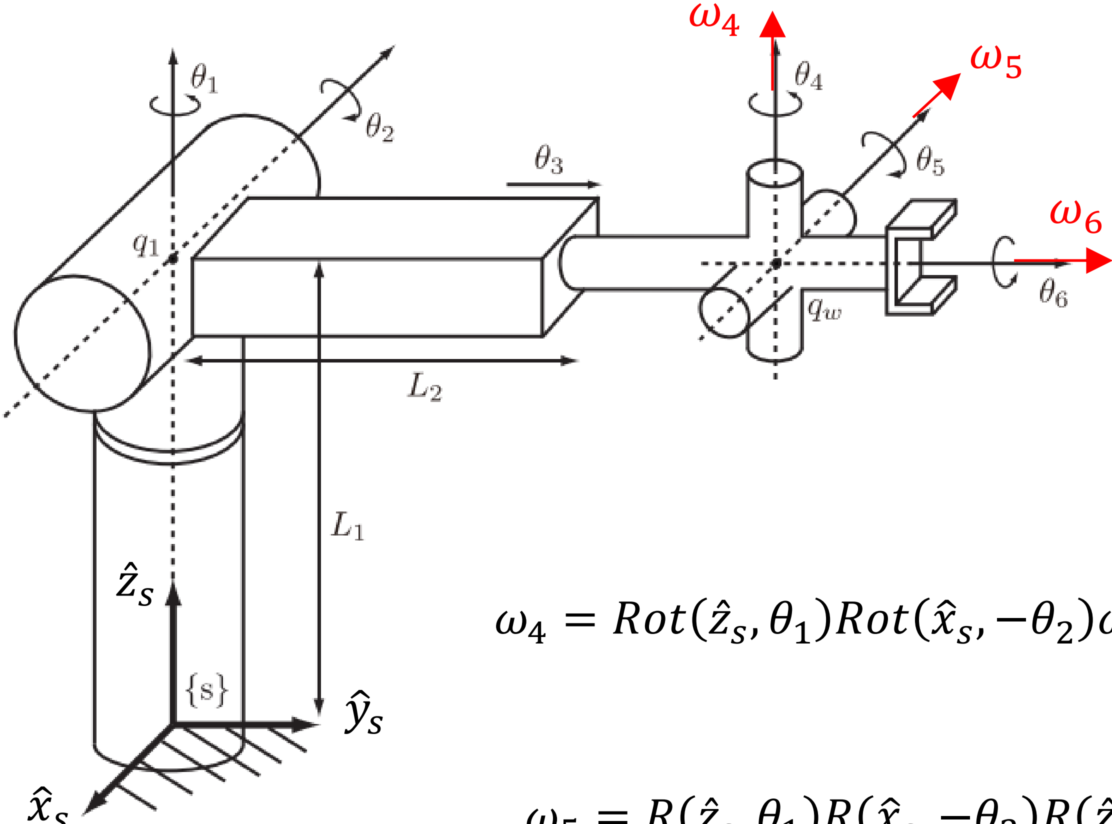
Alternative we can perform rotations of θ_1 and θ_2 on the initial position (zero configuration) of v_3. Note v_3 is along $\hat{y}_s$ in zero configuration.

$$v_3^0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

θ_2 rotates v_3^0 around $-\hat{x}_s$- axis (or around $\hat{x}_s$ in negative direction according to RHR). θ_1 rotates v_3^0 around $\hat{z}_s$- axis. We visualize the order of rotation around the axes in {s} and use pre-multiplication.

$$v_3 = Rot(\hat{z}_s, \theta_1) Rot(\hat{x}_s, -\theta_2) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -s_1 c_2 \\ c_1 c_2 \\ -s_2 \end{bmatrix}$$

Space Jacobian E.g.2: Spatial RRPRRR chain



Source: Modern Robotics

Axes of joints 4, 5 and 6 intersect at the same point. They are the 3 dof of the wrist joint to provide the orientation of the end-effector.

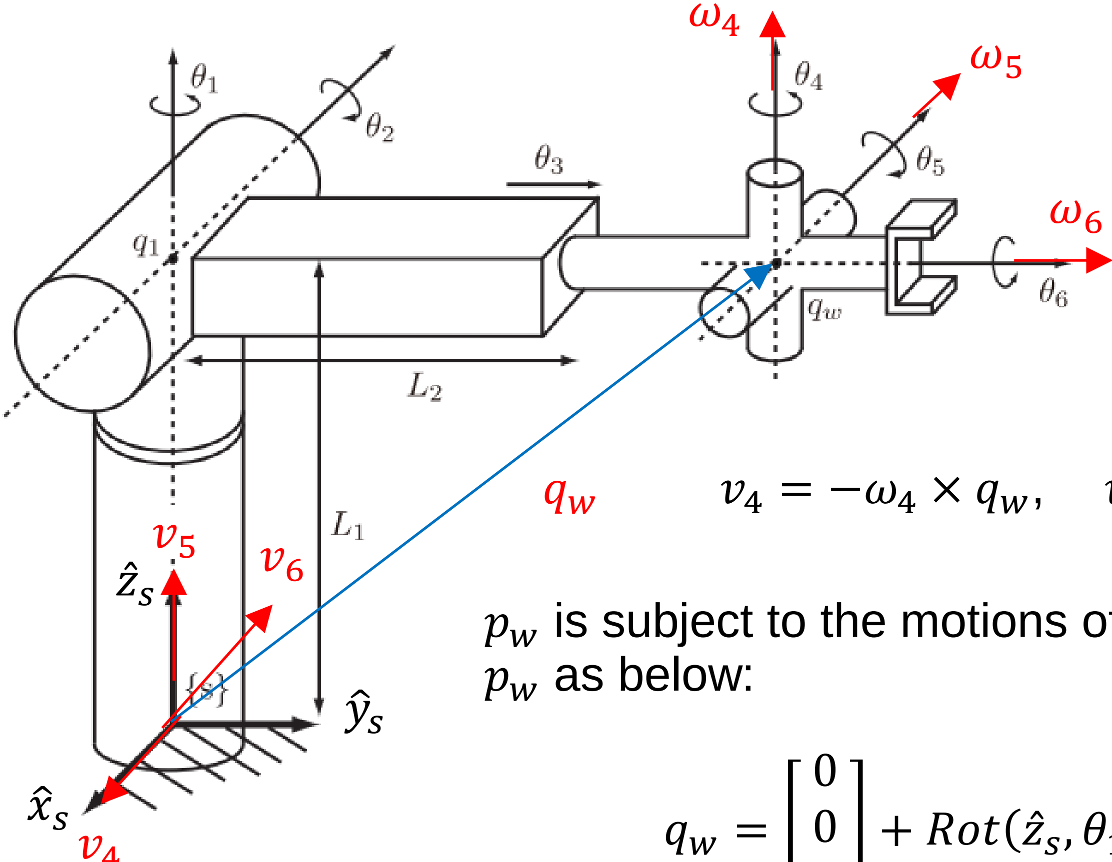
We can obtain ω_4, ω_5 and ω_6 by performing rotations of the joints before each of them.

$$\omega_4 = Rot(\hat{z}_s, \theta_1)Rot(\hat{x}_s, -\theta_2)\omega_4^0 = R(\hat{z}_s, \theta_1)R(\hat{x}_s, -\theta_2)\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\omega_5 = R(\hat{z}_s, \theta_1)R(\hat{x}_s, -\theta_2)R(\hat{z}_s, \theta_4)\omega_5^0 \text{ where } \omega_5^0 = \begin{bmatrix}-1\\0\\0\end{bmatrix}$$

$$\omega_6 = R(\hat{z}_s, \theta_1)R(\hat{x}_s, -\theta_2)R(\hat{z}_s, \theta_4)R(\hat{x}_s, -\theta_5)\omega_6^0 \text{ where } \omega_6^0 = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

Space Jacobian E.g.2: Spatial RRPRRR chain



Source: Modern Robotics

We can obtain v_4, v_5 and v_6 from the cross product of their respective screw axis ω_4, ω_5 and ω_6 with the vector from {s} to the wrist center, q_w.

Considering RHR, we have

$$v_4 = -\omega_4 \times q_w, \quad v_5 = -\omega_5 \times q_w, \quad v_6 = -\omega_6 \times q_w$$

p_w is subject to the motions of joint 1, 2 and 3, we can obtain p_w as below:

$$q_w = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix} + Rot(\hat{z}_s, \theta_1) Rot(\hat{x}_s, -\theta_2) \begin{bmatrix} 0 \\ L_2 + \theta_3 \\ 0 \end{bmatrix}$$

Body Jacobian: by screw

- Let the forward kinematics of an n-link open chain be expressed in the following product of exponentials form:
$$T(\theta) = Me^{[\mathcal{B}_1]\theta_1} \dots e^{[\mathcal{B}_n]\theta_n}$$
- The body Jacobian $J_b(\theta) \in \mathbb{R}^{6\times n}$ relates the joint rate vector $\dot{\theta} \in \mathbb{R}^n$ to the spatial twist $\mathcal{V}_s$ via
$$\mathcal{V}_b = J_b(\theta)\dot{\theta}$$
- The ith column of $J_b(\theta)$ can be found by
$$J_{bi}(\theta) = Ad_{e^{-[\mathcal{B}_n]\theta_n} \dots e^{-[\mathcal{B}_{i+1}]\theta_{i+1}}}(\mathcal{B}_i)$$
- for $i = n-1, \cdots, 1$, with the last column $J_{bn} = \mathcal{B}_n$.
- The i th column $J_{bi}(\theta)$ is the **screw vector for joint axis *i*, expressed in the body (end-effector) coordinate frame**, as a function of the joint variables $\theta_{i-1}, \cdots, \theta_n$.

Summary (1/2)

- Velocity kinematics is about determining the twist of the end-effector given the velocities and positions at the joints.
- Jacobian is the linear sensitivity of the end-effector velocity to the joint velocity $\dot{\theta}$.

$$tip\ velocity\ v_{tip} = J_1(\theta)\dot{\theta}_1 + \cdots + J_n(\theta)\dot{\theta}_n$$

- Jacobian matrix maps the joint velocity space to the end-effector velocity space.
- Jacobian can map a unit spherical joint velocity boundary to the end-effector manipulability ellipsoid.
- The manipulability ellipsoid can quantify closeness of a configuration to singularity.

Summary (2/2)

- Spatial Jacobian uses similar screw axis (expressed in space frame) technique in power of exponentials (forward kinematics) to determine the columns of a Jacobian matrix.
- Body Jacobian uses similar screw axis (express in body frame) technique in power of exponentials (forward kinematics) to determine the columns of a Jacobian matrix.
- Jacobian matrix will be used in solving inverse kinematics problems.

Reading List

- Read Chapter 5.1 of Modern Robotics

To Do List

- Watch Chapter 5 videos of Modern Robotics on Coursera, or on YouTube https://www.youtube.com/playlist?list=PLggLP4f-rq02vX0OQQ5vrCxbJrzamYDfx