Tutorial 2 – Sample Solution Digital Logic

CO 2206 Computer Organization

Logic Gate ICs – Ans.

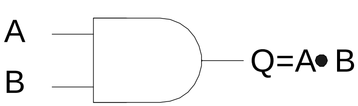
- **Task 1:** Determine (research) a part number of the IC for the following logic gates:
  - 2-input NAND gates: 7400 (can be any series/manufacturer) or CD4011
  - 3-input NAND gates: 7410 or CD4023
  - 2-input NOR gates: 7402 or CD4001
  - 3-input NOR gates: 7427 or CD4025
  - NOT gates: 7404 or CD4049
  - OR gates: 7432 or CD4071
  - AND gates: 7408 or CD4073, CD4081
  - XOR gates: 7486, 74136, 74386

Boolean Algebra – 1 – Ans.1

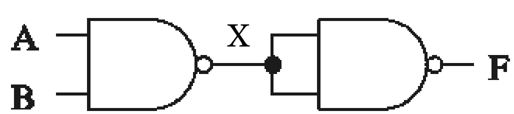
- **Task 2**: Solve the following problems
  - Prove the *absorption law* using *Boolean Algebra:* x·(x+y) = x

x·(x+y)	= (x·x) + (x·y)	distribution law
	= x + (x·y)	idempotence law
	= (x.1) + (x·y)	identity law
	= x·(1 + y)	distribution law
	= x·1	null law
	= x	identity law

  - Prove the implementations of *AND*, *OR* and *NOT* using *NAND*
    - Prove above using *Truth Table* and *Boolean Algebra*



A	B	Q
0	0	**0**
0	1	**0**
1	0	**0**
1	1	**1**



A	B	X	F
0	0	1	**0**
0	1	1	**0**
1	0	1	**0**
1	1	0	**1**

F=(X.X)'
=X' idempotent
=(A.B)''
=A.B double negation

Boolean Algebra – 1 – Ans.2

– Prove equivalence, $(xyz)' = x' + y' + z'$ by means of *truth table*

x	y	z	xyz	(xyz)'	x'	y'	z'	x'+y'+z'
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	0	0	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

Boolean Algebra – 1 – Ans.3

– Prove equivalence by means of *Boolean algebra:*

- $(x+y)'\cdot(x'+y')' = 0$
- $xy = (x+y)(x+y')(x'+y)$
- $xy + x'z = xy + x'z + yz$

$(x+y)'\cdot(x'+y')'$	$= (x'\cdot y')\cdot(x\cdot y)$	de Morgan
	$= (x'\cdot y'\cdot x)\cdot y$	associative
	$= [(x'\cdot x)\cdot y']\cdot y$	commutative
	$= (0\cdot y')\cdot y$	idempotent
	$= 0\cdot y = 0$	null, null

$(x+y)(x+y')(x'+y)$	$= (x+y)(x+y)(x+y')(x'+y)$	idempotent
	$= \mathbf{(x+y)(x+y')}(x+y)(x'+y)$	associative
	$= \mathbf{[xx+xy'+xy+yy']}[xx'+xy+yx'+yy]$	distribution
	$= [x+x(y+y')+0][0+y(x+x')+y]$	idempotent, distribution, complement
	$= (x+x.1)(y.1+y)$	complement
	$= (x+x)(y+y)$	identity
	$= xy$	idempotent

$xy+x'z+yz$	$= xy + x'z + yz(x + x')$	complement and identity
	$= xy + x'z + xyz + x'yz$	distribution
	$= xy(1+z) + x'z(1+y)$	distribution
	$= xy + x'z$	null

Boolean Algebra - 2

- **Task 3**: Solve the following problems
  - Demonstrate by means of truth tables the validity of the following identity:
    - XOR's inverse: $(x \oplus y)' = x \oplus y'$
  - Prove the identity of each of the following *Boolean equations*, using algebraic manipulation:
    - Prove $A'B' + AB = (A \oplus B)'$. Hint: use double negation
    - $(a \oplus b)' \oplus c = a'b'c' + abc' + a'bc + ab'c$
  - Simplify the following logic functions using *Boolean algebra* rules:
    - $ab + ab'$
    - $xyz + x'y + xyz'x' + xy + xz' + xy'z'$
  - Obtain the truth table of the function $(xy + z)(y + xz)$ and express the function in *sum-of-minterms* and *product-of-maxterms*

Boolean Algebra – 2 – Ans.1

x	y	x ⊕ y	**(x ⊕ y)'**	x	y'	**x ⊕ y'**
0	0	0	**1**	0	1	**1**
0	1	1	**0**	0	0	**0**
1	0	1	**0**	1	1	**0**
1	1	0	**1**	1	0	**1**

A'B' + AB = (A'B' + AB)'' double negating
= ((A'B')'(AB)')' de Morgan on inner negation
= ((A+B)(A'+B'))' de Morgan
= (AA' + AB' + A'B + BB')' distribution
= (0 + AB' + A'B + 0)' complement
= (A ⊕ B)' XNOR

(a ⊕ b)' ⊕ c
= (a ⊕ b)'c' + (a ⊕ b)c XOR: x ⊕ y = x'y + xy'
= (a ⊕ b')c' + (a ⊕ b)c XOR inverse: (x ⊕ y)' = x ⊕ y'
= (ab + a'b')c' + (ab'+a'b)c XOR: x ⊕ y = x'y + xy'
= abc' + a'b'c' + ab'c + a'bc distributive
= a'b'c' + abc' + a'bc + ab'c associative

ab + ab'
= a(b+b') distributive
= a.1 complement
= a identity

Boolean Algebra – 2 – Ans.2

xyz + x'y +xyz'x' + xy + xz' + xy'z'
= xyz + x'y + yz'.0 + xy + xz' + xy'z' complement: xx' = 0
= xy(z+1) + x'y + xz'(y+1) null: yz'.0 = 0; distributive ab+a = a(b+1)
= xy + x'y + xz' null
= y(x+x') + xz' distributive
= y + xz' complement

Let F = (xy+z)(y+xz)

x	y	z	minterms	maxterms	xy	xz	xy + z	y + xz	F
0	0	0	m0 = x'y'z'	M0 = x+y+z	0	0	0	0	0
0	0	1	m1 = x'y'z	M1 = x+y+z'	0	0	1	0	0
0	1	0	m2 = x'yz'	M2 = x+y'+z	0	0	0	1	0
0	1	1	m3 = x'yz	M3 = x+y'+z'	0	0	1	1	1
1	0	0	m4 = xy'z'	M4 = x'+y+z	0	0	0	0	0
1	0	1	m5 = xy'z	M5 = x'+y+z'	0	1	1	1	1
1	1	0	m6 = xyz'	M6 = x'+y'+z	1	0	1	1	1
1	1	1	m7 = xyz	M7 = x'+y'+z'	1	1	1	1	1

F(x,y,z) = Σm(3,5,6,7) ;sum of minterms are sum of m terms when F=1
F(x,y,z) = ΠM(0,1,2,4) ;product of maxterms are product of M terms when F=0